Seller Collusion in Two-Sided Markets

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Abstract

This paper studies how a platform's fee structure shapes seller collusion in two-sided markets. I show that higher per-unit fees can sustain collusion by making deviation less attractive. As this allows platforms with a tool to influence seller behavior, platforms may have an incentive to limit competition: colluding sellers extract surplus from buyers, which the platform can capture through its fee. But this creates a trade-off: higher fees encourage collusion, yet may also reduce buyer demand. The optimal fee structure depends on network effects, with stronger effects making collusion less appealing. As a result, platforms may need to adjust fees downward to foster competition, which can be undesirable for them. Established platforms with a strong buyer base, and thus weaker network effects, are therefore more prone to enabling collusion than those still growing.

JEL Classification codes: D40, L10, L40

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. All remaining errors are solely mine.

1 Introduction

In today's increasingly digital economy, online marketplaces provide consumers with access to an expanding range of products and services. Examples range from e-commerce marketplaces like Amazon and eBay to accommodation websites like Airbnb or Booking.com, to mobile app stores, and many more. Such multi-sided marketplaces add value to sellers by attracting many consumers beyond local markets. Similarly, consumers may benefit from greater product variety. This gives rise to positive indirect network effects: a growing number of buyers attracts more sellers and vice versa.

Given the growing economic relevance of online marketplaces, it is unsurprising that the number of high-profile antitrust cases involving anti-competitive seller behavior has surged (see OECD, 2018).[1] Additionally, the use of price-matching algorithms is now fairly established in digital marketplaces, and their growing popularity is likely to amplify this number (Calvano et al., 2020; Ezrachi and Stucke, 2019; CMA, 2021).[2] However, existing legal frameworks are often ill-adapted to counteract their anti-competitive consequences, and many regulators and supranational bodies express concerns about the resulting tacit "agreements" among sellers (OECD, 2017; Gal, 2023; Bernhardt and Dewenter, 2020; Gal, 2019; Ezrachi and Stucke, 2019).[3] Overall, this suggests that the number of actual cases is far greater than those that have been prosecuted so far.

In this paper, I investigate how a platform's fee choice affects sellers' incentives to collude and whether a platform has an incentive to correct collusive seller behavior. Using a game-theoretic model, I find that a platform can facilitate sellers' collusion incentives by increasing its fee. By doing so, the platform makes defecting from the collusive regime less attractive and hence encourages seller collusion. Importantly, my model assumes that a platform cannot choose its fees flexibly, which is a restriction I impose to keep the analysis

[1] There are numerous examples of seller collusion in two-sided markets. For instance, several sellers in the DVD and Blu-Ray segment on Amazon recently pleaded guilty to price-fixing charges by the US Department of Justice (DoJ) (2022). Similarly, the Italian Competition Authority (AGCM) (2020) fined resellers in the earphone segment on Amazon. Other well-known cases include the use of pricing algorithms to reduce competition, such as in the posters and picture frames market, investigated by the British Competition and Markets Authority (CMA) (2016), and the DoJ (2015; 2016). The CMA (2018) also found widespread use of algorithms to set prices, particularly on online platforms.

[2] Some scholars distinguish between collusion facilitated by a dedicated code (collusion by code) and collusion resulting from algorithmic pricing (algorithmic collusion). For simplicity, I treat both as instances of algorithmic collusion in this paper.

[3] The current legal inaptitude stems from the fact that traditional antitrust laws focus on detecting explicit collusion, whereas algorithmic pricing leads to automated, tacit coordination that unfolds without overt communication, making it inherently more elusive to existing legal standards (see, e.g., Gal, 2023; Bhadauria and Vyas, 2018).

tractable. Yet, although this may seem strict at first, it is nonetheless reasonable: in practice, platforms usually have a myriad distinct product categories, so changing their fees category by category constitutes an enormous administrative burden for them, suggesting that platforms are indeed less flexible in setting their fees.[4]

The main finding of this paper is that, depending on how sellers discount future profits, a platform can have an incentive to limit competition. This is especially true when network effects are sufficiently weak or a greater number of buyers remain active on the platform in case of seller collusion. In fact, when the platform employs a per-unit fee, it faces a trade-off between extracting surplus via its fee and maximizing transactions in its marketplace. Thus, when the surplus per interaction increases, network benefits increase as well, so the total number of transactions becomes more sensitive to changes in prices. This makes higher prices charged by colluding sellers less desirable for the platform. Conversely, when the number of transactions remains relatively inelastic with respect to price changes, colluding sellers can capture substantial surplus from the buyer side, which the platform can then extract through its fee. Thus, in the case of seller collusion, the platform behaves like a traditional monopolist.

However, the platform may be constrained in setting its optimal fee if it seeks to promote a competitive marketplace: as large fees induce seller collusion, the platform may be forced to choose a level of fees that would be otherwise too low in order to counteract this "encouragement effect" on sellers. Therefore, since a platform that wants to induce competition may need to adjust its fees downwards, it becomes less desirable to encourage competition. However, the potential to foster seller collusion through the fee decreases when network effects are particularly strong, as they increase profits.

The paper is organized as follows. Section 2 situates the findings within the broader literature. Section 3 introduces the baseline model, which includes three types of agents: buyers, sellers, and a monopoly platform. Sellers provide horizontally differentiated goods, while the platform sets per-unit fees, which only apply to the seller side. I discuss the relevance of per-unit fees, which are commonly used in the platform economy, in Appendix 2. Sections 4 to 6 analyze the game: Section 4 examines seller interactions and collusion incentives, showing that higher fees may encourage collusion by reducing the incentives to deviate from collusive agreements. Section 5 explores buyer entry decisions, revealing that higher platform fees can reduce buyer demand as higher fees might be passed on to buyers. Section 6 analyzes the platform's optimal fee structure and preferred seller conduct, finding that the

[4] For instance, Amazon rarely adjusts its fees, even though economic circumstances may change a lot (see, e.g., Shopkeeper, 2025; Seller Snap, 2025). Likewise, Airbnb kept its fee structure unchanged since the COVID-19 pandemic (Hostaway, 2025). Additionally, frequent changes in platform fees would raise antitrust agencies' suspicion as it might indicate collusion among platforms.

platform may extract all surplus when promoting collusion but may be constrained by the risk of inducing collusion if it seeks to foster competition. The platform's preferred conduct is influenced by the strength of indirect network effects, which could make collusion more favorable, particularly for mature platforms with a large user base. Section 7 discusses other fee structures, such as revenue-sharing, membership, and profit-sharing, noting that my results on per-unit fees do not extend to membership and profit-sharing fees. Section 8 offers policy implications, while Section 9 concludes.

2 Related literature

This paper contributes to various strands of the literature in industrial organization studying multi-sided markets.

Platform pricing. A substantial part of the literature studying multi-sided markets focuses primarily on the interplay between a platform's underlying network effects and pricing decisions (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Armstrong and Wright, 2007; Weyl, 2010). Typically discussed pricing models in this literature include *per-unit* (or transaction) fees and *fixed membership* fees; rarely also *royalty* fees (also called ad valorem or revenue-sharing fees) or *two-part tariffs*. Building on these theories, I additionally investigate how platform pricing affects sellers' ability and their incentives to collude.

Platform governance. Besides pricing aspects, this paper also contributes to the more recent literature that studies platform governance decisions (see, e.g., Boudreau, 2010; Parker and Van Alstyne, 2018; Hagiu and Spulber, 2013; Edelman and Wright, 2015; Hagiu and Wright, 2019; Teh, 2022; Schlütter, 2024; Gomes and Mantovani, 2025; Johnen and Somogyi, 2024; Rhodes et al., 2023; Belleflamme and Johnen, 2024). Most of these works examine the implications of innovation on platforms (Boudreau, 2010; Parker and Van Alstyne, 2018; Hagiu and Spulber, 2013; Edelman and Wright, 2015; Hagiu and Wright, 2019). Teh's paper (2022) comes closest to this by compellingly examining how a platform's fee structure may affect seller competition by controlling sellers' markups. Instead, I significantly extend these insights and study how fees affect seller collusion.

Schlütter (2024) studies the effect of *price parity clauses* (PPCs) and seller collusion on intermediary platforms, abstracting from network effects. While PPCs are not my focus, my model similarly normalizes sellers' outside options to zero, which can be interpreted as sellers expecting equal profits across all distribution channels. In the paper by Johnen and Somogyi (2024), platforms use price transparency to manage network effects. Instead, I explore how platform fees affect seller pricing by inducing collusion or competition.

Managing seller competition. Due to its overlap with the literature on platform governance, my work also relates to the literature that studies how platforms manage competition among sellers (Belleflamme and Peitz, 2019; Anderson and Bedre-Defolie, 2021; Padilla et al., 2022; Nocke et al., 2007; Hagiu, 2009; Teh, 2022; Schlütter, 2024). In most of these papers, the number of sellers on the platform is determined endogenously by the platform's pricing decision, which, in turn, can be affected by other exogenous factors (such as cross-group network effects or consumer preferences). For instance, Belleflamme and Peitz (2019) and Karle et al. (2020) study platforms imposing membership fees and their consequences on the number of sellers (and thereby the degree of competition between sellers) without modeling seller competition explicitly. Closely related, Edelman and Wright (2015), Hunold et al. (2018), and Schlütter (2024) abstract from network effects to explore PPCs restricting sellers from charging lower prices elsewhere than via the platform. In my paper, the number of sellers is fixed, but fees affect sellers' collusion incentives.

Algorithmic collusion on platforms. As the potential harm of colluding sellers in online marketplaces grew, another strand of the literature emerged that focuses on seller collusion in multi-sided markets based on the use of price-matching algorithms (Calvano et al., 2020, 2021; Klein, 2021; Miklós-Thal and Tucker, 2019; Ezrachi and Stucke, 2019; Hansen et al., 2021; Rhodes et al., 2023). In a set of simulation studies, for instance, Calvano et al. (2020, 2021) find that when sellers use algorithms to price their products, tacit collusion is almost certain to arise, independent of cost or demand asymmetries, the number of sellers, and uncertainty Moreover, such cartels remain stable over time, even though seller algorithms have not been initially trained or instructed to sustain collusion.

In a more recent paper, Rhodes et al. (2023) investigate algorithmic collusion, theoretically and experimentally, in a similar setting. Their primary question revolves around whether elevating the prominence of specific sellers can diminish price coordination among them. Their findings reveal that platforms can indeed disrupt seller cartels, even when sellers are infinitely patient. However, my article diverges from theirs in one fundamental way: I extend the inquiry beyond this aspect and study whether a platform possesses an incentive to promote competition within its marketplace actively.

Most of these works, however, neglect network effects between buyers and sellers. Thus, their focus lies on the feasibility of seller cartels without direct coordination among the colluding parties. Instead, my work offers a theoretical perspective on seller collusion and highlights mechanisms in which a platform can encourage seller collusion by making use of indirect network effects and its fee. Specifically, I show that platforms can be motivated, in the sense of a *hub-and-spoke* cartel (Ezrachi and Stucke, 2019), to establish some form of price coordination among sellers when buyers' willingness to pay is sufficiently large.

Regulating online platforms. Linked to the literature on algorithmic collusion, there is growing scrutiny on the regulatory side about harmful commercial practices by established online platforms (often called tech giants or gatekeepers).[5] Such practices include, e.g., (potential) *killer acquisitions* (Hemphill, 2020; Cunningham et al., 2021; Motta and Peitz, 2021) as suspected after Meta's acquisition of Giphy,[6] *self-preferencing* by Amazon and Google (i.e., favoring their own products over third-party seller products on their marketplaces) (Hagiu et al., 2022), *predatory pricing* by Amazon (Khan, 2016), or *misleading sales tactics* by Booking.com to put pressure on consumers (Teubner and Graul, 2020). I contribute to this debate by stressing how network effects may incentivize a platform to exploit its dominance in pricing and regulating its marketplace to encourage seller collusion.

Vertically related markets and collusion. Finally, the literature on vertical contracts and downstream collusion offers insights into the dynamics of collusion within supply chains, secret vertical contracts between retailers and suppliers, or vertical integration considerations. For instance, Piccolo and Miklós-Thal (2012) find that downstream firms with buyer power can more easily engage in collusion when they cooperate on input supply contracts, with specific contract characteristics facilitating collusion. Gilo and Yehezkel (2020) emphasize that vertical collusion involving secret vertical contracts can be easier to sustain than collusion among retailers themselves, addressing commitment problems and incentivizing adherence through slotting allowances.

Further, Bonanno and Vickers (1988) stress the advantages of vertical separation over vertical integration, specifically in terms of inducing more friendly behavior from rival manufacturers and facilitating collusion. However, my paper differs from the existing literature in one key way. In fact, while a monopolistic wholesaler in a traditional vertical market may facilitate collusion, I show that a two-sided platform may instead prefer competition, depending on the strength of network effects. The key distinction is that the platform intermediates between sellers and buyers, while a traditional wholesaler –lacking network effects– typically maximizes rent extraction with seller collusion.

3 Model

To understand how a platform's pricing decisions may shape seller competition and how the platform may benefit from it, I propose a model in the spirit of Armstrong (2006), Johnen and Somogyi (2024), or Teh (2022) with three types of agents: a monopoly platform, sellers, and

[5] See, e.g., New York Times (2020). An example of regulators' concerns about, and implemented actions against, such practices can be found in, e.g., the *Digital Markets Act* by the European Commission (2022).

[6] See, e.g., Financial Times (2021).

buyers. Sellers provide horizontally differentiated products, and buyers and sellers interact exclusively via the platform. The platform is assumed to levy a per-unit fee that is imposed only on the seller side.[7] I first sketch out the game and its timeline before explaining it more thoroughly.

3.1 Overview and stages

The model and timeline can be summarized as follows. There is a common discount factor $\delta \in (0, 1)$ for all agents. In the first stage of the game, the platform sets its per-unit fee.

In the second stage, buyers and sellers decide whether to join the platform or not. They join as long as they can generate a positive benefit from being active on the platform. Importantly, buyers enjoy a stand-alone benefit $a > 0$ from being active. To simplify the analysis, I assume that there is only one product category with potentially two sellers.[8]

The third stage involves an infinitely repeated game with sellers who offer horizontally differentiated goods. Sellers set prices and decide whether to compete or to collude. Their marginal costs are equal to $c > 0$. I model the product space as the unit interval, with each seller's product corresponding to a location at the endpoints of the unit interval. I denote the product differentiation parameter by $\tau > 0$. If sellers collude, they coordinate on charging the monopoly price.[9] Alternatively, competing sellers interact in a stylized Hotelling fashion (Johnen and Somogyi, 2024; Heidhues and Kőszegi, 2018, 2017; Bénabou and Tirole, 2016). Buyers who join the platform draw their position on the Hotelling line and make their purchase decisions. I denote their willingness to pay by $v > 0$. Thus, while the first stage captures the monopolist's platform pricing decision, the second stage features user entry decisions, and the third stage models interactions between sellers on the platform. I solve the model by using subgame perfection.[10] The following subsections describe the model in more detail.

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[7] In Appendix 2, I argue about the relevance of per-unit fees in the digital economy. However, I also discuss results on other types of fee structures in Section 7.

[8] Note that while my model features only one product category, it can be extended to a platform with infinitely many product categories. In fact, my results remain qualitatively unchanged in an analysis with multiple product categories as long as they are identical and independent from each other, as competitive interactions and incentives remain the same across product categories, and so the platform charges the same fees across all product categories.

[9] In principle –and as suggested by the Folk Theorem (Friedman, 1971)– colluding sellers could charge any price between the competitive price and the monopoly price. Following the intuition by Motta (2004), I show further below that my results remain robust if sellers charge a different price.

[10] Moreover, when there is a multiplicity of equilibria, I focus on the collusive outcome where sellers charge the monopoly price.

3.2 Stage 1: Platform pricing

In the first stage, the platform sets a per-unit fee f.[11] As the third stage involves an infinitely repeated game, the platform sets f for all periods $t = \{0, 1, 2, \dots\}$ of the third stage (see below). I focus on a retail platform that only charges the seller side. Crucially, the choice of the fee f will influence seller interactions and therefore also users' entry decisions. The platform has no marginal cost for serving its users.

3.3 Stage 2: Buyers' and sellers' entry decisions

In the second stage, there are potentially two sellers and infinitely many buyers who decide to join the platform or not. The valuations of sellers v^S and buyers v^B from joining the platform are:

$$v^S(f) = \sum_{t=0}^{\infty} \delta^t * \pi_t(f) * \frac{1}{2} N^B(f) \quad \text{and} \quad v^B(f) = \sum_{t=0}^{\infty} \delta^t \left(a + u_t(f) * \frac{1}{2} N^S(f) \right), \tag{1}$$

where $N^B(f)$ denotes the mass of buyers and $N^S(f)$ the number of sellers on the platform, respectively. The parameter $a > \tau/4$ represents a stand-alone utility that buyers enjoy when joining the platform.[12] $\pi_t(f)$ and $u_t(f)$ are the sellers' expected per-period profit per interaction and the buyers' expected per-period benefit per interaction, respectively.

These valuations reflect two common assumptions. First, sellers interact with $N^B(f)$ buyers on the platform and obtain an expected per-period benefit $\pi_t(f)$ per buyer on the platform. Likewise, each buyer interacts with one of the N^S sellers, resulting in a per-interaction benefit of $u_t(f)$ per period on the platform. Consequently, buyers benefit from more sellers and vice versa: the platform features positive cross-group externalities. Second, the marginal value of each additional interaction is constant for buyers and sellers on the platform.

I assume that there is a mass one of potential buyers whose outside option is heterogeneous and uniformly distributed along the unit interval. As mentioned above, there are two potential sellers. To simplify the analysis, I assume that the sellers' outside option is homogeneous and normalized to zero (see, e.g., Johnen and Somogyi, 2024). Thus, all sellers are subject to the same outside option. In particular, a homogeneous outside option

[11] Other fees, including membership fees along with profit- and revenue-sharing fees, are discussed in Section 7.

[12] For simplicity, I assume a is fixed for all buyers. However, the analysis would still hold if a follows any distribution, as long as there are mass points where $a > \tau/4$. The key requirement is that $a > \tau/4$ in some parts of the unit interval, ensuring that some buyers will join the platform even when sellers charge $p^{col} = v$ (as in the Hotelling model, the per-period buyer surplus is $v - p_t - \tau/4$, and $N^B(p_t = p) = a - \tau/4 + v - p$ in equilibrium).

for sellers implies that more buyers increase the benefits for sellers $v^S(f)$ without attracting more sellers in equilibrium.[13] This limits the positive cross-group externalities without fully eliminating them, yielding a simplified demand system.

Importantly, and as indicated above, the platform can influence the per-interaction benefits by governing competition on the seller side with the fee f it imposes on them. To capture this, I endogenize $\pi_t(f)$ and $u_t(f)$ by modeling interactions between buyers and sellers explicitly in the third stage, based on a model of spatial seller competition on the platform (Teh, 2022; Johnen and Somogyi, 2024).

3.4 Stage 3: Seller interactions

As the primary focus of this paper is on how the platform can affect sellers' attitudes toward tacit collusion in terms of prices, the third stage features an infinitely repeated game with periods denoted by $t \in \{0, 1, 2, \dots\}$ and 2 sellers competing in a stylized Hotelling fashion (see below). Sellers are symmetric with marginal costs c and each of them is located at the endpoints of the Hotelling line of length one. They offer horizontally differentiated products to buyers who purchase at most one unit. I denote each seller (and their location in the product space) by $i \in \{0, 1\}$, with the product space being the continuum $[0, 1]$.

Buyers who entered the platform draw in each period t of the third stage their location on the Hotelling line and are uniformly distributed on it. A buyer located at $x \in [0, 1]$ incurs a disutility of $\tau|x - i|$ when buying from seller i, where τ is a product differentiation parameter that determines a seller's market power. A buyer has a valuation v for purchasing the product, while the outside option provides a gross utility of 0. Following Heidhues and Kőszegi (2018), I assume that the outside option is available only at the endpoints of the unit interval, so that a buyer located at $x \in [0, 1]$ incurs a disutility of $\tau \min\{x, 1 - x\}$ when forgoing a purchase from one of the sellers.[14]

[13] More precisely, homogeneity of the seller outside option implies that the number of sellers remains unchanged as long as seller profits exceed the value of that outside option.

[14] Unlike in the standard Hotelling (1929) model, where the product differentiation parameter τ influences both competitive intensity and the relative attractiveness of the purchase decision, this specification isolates the effect of τ solely for strategic interactions among sellers. Consequently, buyers choose to purchase from seller i provided that $v \geq p_t^i(f)$, which implies that the collusive (or monopoly) price is given by $p_t^{col} = v$. This rules out cases of constrained monopoly equilibria, where sellers that raise prices lose buyers to the outside option, like a monopolist would, and sellers that lower prices win buyers from the rival, like competing sellers would. As this constrained monopoly case seems particular and ambiguous from a policy perspective (Salop, 1979), I proceed by assuming buyers' outside options are at the seller's location to avoid it in my analysis. Additionally, as $a > \tau/4$, it improves active buyers' outside option, as no participation yields zero utility; participation yields a surplus of at least $a - \tau/4 > 0$. For further applications of this version of the Hotelling model, see Johnen and Somogyi (2024), Heidhues and Kőszegi (2017), and Bénabou and Tirole (2016).

I further assume that v is sufficiently large to ensure that in equilibrium, both sellers join the platform and all buyers make a purchase, leading to full market coverage.[15] All buyers are aware of seller i's price $p_t^i(f)$ in period t. Also, in each period, each seller offers a single, horizontally differentiated product. As indicated above, sellers weigh their future profits with the common discount factor $\delta \in (0, 1)$.

For the sake of simplicity, I assume that sellers use grim-trigger strategies (as in Friedman, 1971) when colluding.[16] Thus, colluding sellers maximize their future stream of per-interaction profits by selecting a price p_t^{col} that jointly maximizes these profits $\pi_t^{col}(f)$. However, as deviating from the collusive agreement may be profitable (e.g., by charging a price $p_t^{dev}(f) < p_t^{col}$, which yields a per-period profit of $\pi_t^{dev}(f)$ per interaction for the deviator), both sellers revert to an eternal punishment phase where they commit to charging the competitive price $p_t^{com}(f)$ for all subsequent periods.

Since such a strategy constitutes a credible punishment against deviations, the sellers' incentive compatibility constraint to abide by the collusive regime becomes

$$\pi_t^{col}(f) + \sum_{t=1}^{\infty} \delta^t \pi_t^{col}(f) \;\geq\; \pi_t^{dev}(f) + \sum_{t=1}^{\infty} \delta^t \pi_t^{com}. \tag{2}$$

for all t. Moreover, since there is no uncertainty, sellers will either compete or collude in each period. As a result, $\pi_t^{col} = \pi^{col}$ and $\pi_t^{com} = \pi^{com}$ for all periods t. Then, by using the geometric series, the expression above can be rearranged to obtain a lower bound for the common discount factor above which collusion is always subgame-perfect:

$$\delta \;\geq\; \frac{\pi^{dev}(f) - \pi^{col}(f)}{\pi^{dev}(f) - \pi^{com}(f)} \;\equiv \delta^*(f). \tag{3}$$

Hence, seller collusion is stable for any $\delta > \delta^*$. Similarly, for any $\delta < \delta^*$, sellers compete for all periods in the third stage, and for any $\delta = \delta^*$, sellers are indifferent between competing or colluding. Importantly, since seller profits per interaction may depend on the particular fee f imposed by the platform (set in the first stage), the platform may be able to influence collusion incentives by raising or lowering its fee f.

[15] In principle, market coverage could depend on the platform's fee f, which is endogenous. However, as I show below, in equilibrium the platform chooses f such that the marketplace will be entirely covered. Additionally, as v is homogeneous across buyers, it implies that differences in preferences only arise due to product differentiation.

[16] Grim trigger strategies imply that a seller i adopts the competitive strategy in all future periods if a competitor deviates from the collusive strategy. While shorter punishment periods are also possible, I focus on infinite punishment to characterize situations where collusion is sustainable. For many values of δ that support collusion, sellers could achieve the same outcome with shorter punishments (see, e.g., Abreu, 1986, 1988; Abreu et al., 1986). It thus implies that my model gives collusion as an outcome the "best chance" for any combination of parameters, such that collusion is actually played out so that it is indeed the equilibrium outcome. I thus focus on collusion whenever collusion is a potential outcome.

3.5 Solution concept and parameter restrictions

I look for subgame-perfect equilibria to solve the model. Starting from the last stage, I study buyers' purchasing behavior given sellers' prices and analyze the infinitely repeated game to identify the critical discount factor necessary for sustaining collusion. In doing so, I highlight how the platform may influence seller collusion incentives by analyzing its effect on this critical discount factor. I then proceed backward to the second stage, where buyers and sellers decide to join the platform, to analyze how equilibrium prices affect user entry decisions. Lastly, I examine the platform's pricing decision in the first stage and carve out conditions for when it prefers which type of seller conduct.

To ensure the model remains well-defined, I impose the following parameter restriction:

$$0 < a - \frac{\tau}{4} \leq v - c - \tau \leq \frac{1-\delta}{2}. \tag{4}$$

The lower bound $0 < a - \tau/4$ guarantees a strictly positive mass of buyers under full collusion, while $a - \tau/4 \leq v - c - \tau$ ensures that buyers still participate in the competitive equilibrium. The upper bound $v - c - \tau \leq (1-\delta)/2$ limits the mass of buyers under competition to at most one. These restrictions together limit the equilibrium mass of buyers to the unit interval and rule out degenerate cases in which either no buyer joins or the market is oversaturated.

4 Seller behavior and collusion incentives

In the third stage, the platform's fee and user entry decisions are predetermined (or fixed), and can therefore be treated as exogenous. I first derive the competitive subgame equilibrium in each period, before identifying the per-period collusive equilibrium in the subgame. To do so, I start by assuming that both δ and δ^* are exogenously drawn by nature, and that δ is either smaller or greater than the critical discount factor δ^*, respectively.

Building on these two equilibria, I then endogenize δ^* by examining how the platform's fee choice influences the selected subgame-perfect equilibrium. I demonstrate that by raising its fee, the platform can strengthen sellers' collusion incentives, ultimately stabilizing collusion.[17]

[17] Lambertini and Sasaki (2001) suggest that higher unit costs can facilitate tacit collusion, although this remains debated. In Appendix 9, I show that higher fees (per-unit or revenue-sharing fees) reinforce collusion incentives whenever demand is scale-invariant. This mechanism thus extends to all models based on market shares rather than absolute demand.

4.1 Competition and collusion

To start, assume first that δ is small enough so that sellers compete in the standard Hotelling fashion ($\delta < \delta^*$). The results in this and the next subsection cover the case where the market is ensured to be entirely covered and that there are no local monopolies on the Hotelling line, so $f \leq v - c - \tau$; the case of $f > v - c - \tau$ is deferred to the Appendix 9.[18] This implies that in each period t, sellers set a price of $p^{com}(f) = c + f + \tau$ and earn a per-interaction profit of $\pi^{com} = \tau/2$ in a symmetric equilibrium:

Lemma 1 (Competitive seller pricing). *$f \leq v - c - \tau$. When $f \leq v - c - \tau$ and sellers compete, there is a unique symmetric equilibrium such that in each period t,*

$$p^{com}(f) = c + f + \tau, \quad \pi^{com} = \frac{\tau}{2}, \quad \text{and} \quad u^{com}(f) = v - p^{com}(f) - \frac{\tau}{4}.$$

All proofs are relegated to Appendix 1. Importantly, when sellers compete, they pass through the per-unit fee f to buyers via the price $p^{com}(f)$. Consequently, while sellers' per-interaction benefit per period π^{com} remains unaffected by f, buyers' per-interaction benefit per period $u^{com}(f)$ decreases as they face larger prices when f increases. The term $\tau/4$ in $u^{com}(f)$ is the average transportation costs that buyers face when going to one of the two sellers.

I now characterize the collusive seller equilibrium when the market is covered, i.e., $f \leq v - c$. Colluding sellers charge a price equal to buyers' willingness to pay v, maximizing per-period per-interaction seller profits while minimizing buyer surplus.[19] To see this, assume $\delta > \delta^*$, so that sellers collude. Since buyers' outside option is located at the sellers' position on the Hotelling line, they purchase in each period t as long as prices p_t^i and p_t^j are below or equal to their willingness to pay v. When sellers coordinate on a collusive price p^{col}, buyers' purchase condition simplifies to $p^{col} \leq v$. Given that their outside option yields a utility of $-\tau/4$ per interaction, the next Lemma characterizes the collusive equilibrium in period t:

Lemma 2 (Colusive seller pricing). *When sellers collude and $f \leq v - c$, there exists a unique symmetric equilibrium such that in each period t,*

$$p^{col} = v, \quad \pi^{col}(f) = \frac{v - c - f}{2}, \quad \text{and} \quad u^{col} = -\frac{\tau}{4}.$$

[18] Results for $f > v - c - \tau$ follow the same logic but lead to different equilibrium outcomes. In particular, if $f > v - c - \tau$, sellers become local monopolies on the Hotelling line, and hence, there would be no difference between collusion and competition in seller behavior. See Appendix 9 for details. As discussed in the final part of this section, however, I show that the platform always chooses a per-unit fee $f^{com} \leq v - c - \tau$ in equilibrium.

[19] I assume sellers coordinate on the joint profit-maximizing price v but could choose any price above unit costs. In Appendix 9, I show that my results hold for all relevant collusive prices. As Motta (2004) argues, lower collusive prices enhance stability, making my results a conservative benchmark for collusion incentives.

Lemma 2 shows that colluding sellers set the monopoly price v, which is independent of the per-unit fee f and maximizes surplus extraction. As a result, buyers are left indifferent between purchasing and their outside option, fully exhausting their willingness to pay. However, since they also incur disutility from purchasing, their net utility in each period t is reduced to the level of their outside option.

4.2 Collusion incentives and stability

I first analyze collusion incentives and stability when sellers jointly maximize future per-interaction profits. I then examine how the platform can influence seller interactions via its fee structure.

As before, I assume sellers use grim-trigger strategies (Friedman, 1971) to sustain tacit collusion: if a seller deviates from the collusive price p^{col}, all sellers revert permanently to the competitive price $p^{com}(f)$. The incentive compatibility constraint for collusion is given by Expression (2), where $\pi_t^{dev}(f)$ denotes the per-interaction profit of a deviating seller. This ensures that the present value of colluding is at least as high as the one-time deviation gain plus the subsequent punishment phase profits.

Using the geometric series and rearranging terms, collusion is subgame-perfect if sellers are sufficiently patient, as shown in Expression (3). Given this constraint, I now derive the optimal deviation strategy for the case of $f \leq v - c - \tau$ and analyze collusion stability. As before, the case of $f > v - c - \tau$ is discussed in the Appendix 9.

Lemma 3 (Deviating seller pricing)**.** *When $f \leq v - c - \tau$ and a seller deviates from the collusive agreement in any given period t, the deviating seller charges a price p_t^{dev} and gets a demand d_t^{dev} such that*

$$p_t^{dev}(f) = \frac{p^{col} + p^{com}(f)}{2}, \; d_t^{dev}(f) = \frac{1}{2} + \frac{p^{col} - p^{com}(f)}{4\tau}, \; \text{and} \; \pi_t^{dev}(f) = \frac{\left(\pi^{col}(f) + \pi^{com}\right)^2}{2\pi^{com}}.$$

Lemma 3 shows that a deviating seller selects a price p_t^{dev} in between p_t^{col} and $p_t^{com}(f)$, the prices of the competitive and collusive equilibria, respectively. Thus, p_t^{dev} is a convex combination between p_t^{col} and $p_t^{com}(f)$, which makes p_t^{dev} a function of f as well. Moreover, since $p_t^{dev}(f) < p_t^{col}$, the deviating seller obtains a market share of $d_t^{dev}(f) > 1/2$ and therefore also larger profits for each interaction.

Based on the results of the Lemmas 1, 2, and 3, I now proceed to the most important result of this section. In particular, the platform may increase sellers' incentives to collude by imposing a larger fee f:

Lemma 4 (Collusion incentives)**.** *The threshold value for sellers' common discount factor $\delta^*(f)$ for future profits to make collusion subgame-perfect decreases monotonically in the platform's per-unit fee f.*

Strikingly, Lemma 4 shows that the platform's fee, f, has a monotonic relationship with the stability of collusion: higher fees make it easier for sellers to sustain collusion by decreasing the threshold value of δ^*, the discount factor required for collusion to be subgame-perfect. To intuitively understand this observation, note first that while for each interaction sellers' (per-period) equilibrium collusive profits $\pi_t^{col}(f)$ and the deviation profits $\pi_t^{dev}(f)$ depend on f, and the (per-period) equilibrium competitive profits π_t^{com} are indepedent of f. Note further that $\pi_t^{col}(f)$ and $\pi_t^{dev}(f)$ are decreasing in f since f enters the profit function similarly to a marginal cost. By Lemma 3, larger fees induce a higher deviation price but also reduce a deviator's market share, making $\pi_t^{dev}(f)$ quadratic in f. Collusive market shares, on the other hand, remain unaffected by f. So, if f increases, $\pi_t^{col}(f)$ and $\pi_t^{dev}(f)$ decrease while π_t^{com} remains unchanged. And since $\pi_t^{dev}(f)$ is quadratic in f while $\pi_t^{col}(f)$ is linear, it decreases faster in f. Therefore, by increasing f, the platform makes deviating from the collusive agreement less attractive, which in turn facilitates seller collusion by reinforcing their incentives to collude. This monotonic relationship is depicted in Figure 1, which shows that the threshold value required for collusion $\delta^*(f)$ is continuously decreasing in f.

5 User entry decisions

The previous section analyzed seller behavior in two different regimes: the competitive equilibrium and the collusive equilibrium. Given sellers' behavior in the third stage, I now discuss user entry decisions in the second stage. In particular, depending on the discount factor δ, I will establish the mass of buyers and sellers who decide to join the platform for an exogenously fixed δ^* to determine their choices in each equilibrium. I first discuss sellers' entry decisions before analyzing buyers' entry choices. As before, this section outlines the results for the case of $f \leq v - c - \tau$ in the competitive equilibrium and $f \leq v - c$ in the collusive equilibrium; respective results for $f > v - c - \tau$ and $f > v - c$ are discussed in Appendix 9.

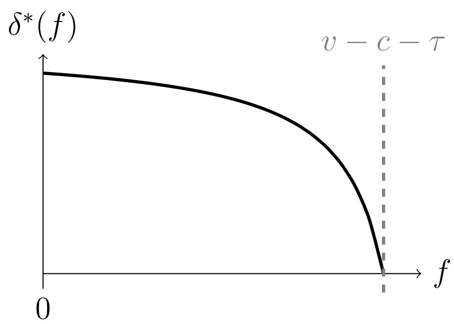


Figure 1: Collusion incentives depending on f.

5.1 Seller entry

Generally, the total number of sellers who join the platform depends on the equilibrium prices set in the third stage, so the number of sellers who enter in equilibrium is given by

$$N^S = N^S\left(p, f, N^B\right) = \begin{cases} 2 & \text{if } f \leq v - c, \\ 0 & \text{else.} \end{cases} \tag{5}$$

Since, by assumption, sellers' outside option to join is homogeneous and normalized to zero, both sellers join the platform as long as they generate non-negative profits. In fact, as the per-unit fee acts like an identical cost-shift for each seller, the entry condition is common to both. So, either both sellers enter or none do, ruling out any single seller equilibrium. I now argue that in both the competitive and the collusive equilibrium, both sellers join.

When $f \leq v - c - \tau$ and sellers compete, they earn a strictly positive per-interaction profit of $\pi_t^{com} = \tau/2$, which is larger than their outside option of zero, so they join the platform. Consequently, $N^S(p^{com}(f)) = 2$ in the equilibrium of the competitive subgame.[20]

In contrast, when $f \leq v - c$ and sellers collude, they can obtain a profit of $\pi_t^{col}(f) = (v - c - f)/2$ per interaction, which is non-negative. Consequently, the number of sellers in the equilibrium of the collusive subgame is given by $N^S = 2$.

5.2 Buyer entry

Different to sellers, buyers have heterogeneous outside options along the unit interval; they prefer lower prices and a greater presence of sellers on the platform. Thus, the number of buyers is given by $N^B = \max\{0, v^B\}$. Recall that due to indirect network effects on the buyer side, v^B is a function of N^S, as indicated in Expression (1). Therefore, in case sellers compete and $f \leq v - c - \tau$, $N^S = 2$, buyers face a price of $p^{com}(f) = c + f + \tau$, so[21]

$$v^B(p^{com}(f)) = \frac{1}{1-\delta}\left(a + u^{com}(f)\right) = N^B(p^{com}(f)). \tag{6}$$

Importantly, since $N^B(p^{com}(f))$ is downward sloping in f, larger platform fees crowd out buyer demand in the competitive subgame equilibrium.

When sellers collude, on the other hand, they charge a price $p^{col} = v$ for all periods t if $f \leq v - c$, so the number of buyers in the collusive subgame equilibrium is

$$N^B(p^{col}) = v^B(p^{col}) = \frac{1}{1-\delta}\left(a + u^{col}\right). \tag{7}$$

[20] In Appendix 9, I show that $N^S = 2$ also for any other $f \leq v - c$ when seller compete. In particular, $f \in (v - c - \tau, v - c]$ implies that prices will be determined by a corner solution, and so sellers become local monopolists on the Hotelling line. For $f > v - c$, the resulting price will be above buyers' willingness to pay, and so buyers stop purchasing.

[21] Since $N^B \geq 0$, I obtain another upper bound for f such that $f \leq v + c + a - 5\tau/4$. However, this constraint does not bind when $a > \tau/4$ (which holds by assumption) and $f \leq v - c - \tau$.

Consequently, buyers enter the platform if and only if $a + u^{col} = a - \tau/4 > 0$, which holds by assumption. The next Corollary summarizes the findings of this section:

Corollary 1 (from Lemmas 1 and 2)**.** *Assume Lemmas 1 and 2 hold. Then the mass of buyers that enter the platform is given by*

$$N^B(p^{com}(f)) = \frac{1}{1-\delta}\,(a + u^{com}(f)) \quad \textit{and} \quad N^B(p^{col}) = \frac{1}{1-\delta}\left(a + u^{col}\right)$$

in case of seller competition and seller collusion, respectively. Moreover, the number of sellers that enter the platform is $N^S(p^{com}(f)) = N^S(p^{col}) = 2$ when sellers compete and collude, respectively.

6 Platform fee design and preferred conduct

The results so far established that a platform can indeed affect seller behavior via its fee f and thereby also user entry decisions. In this section, I first show what fees the platform would set given the different equilibria. I find that in the collusive equilibrium, the platform wants to set the fee as high as possible to maximize surplus extraction. When sellers compete, a trade-off arises: since large fees induce seller collusion, the platform may be constrained in designing its fee that would otherwise be optimal to extract surplus. I then establish the platform's preferred conduct depending on different levels of the discount factor δ.

However, before proceeding, note that with per-unit fees, the platform's discounted profits correspond to

$$\Pi^P(f) = \frac{1}{1-\delta} * \frac{1}{2} * f * N^B(p(f)) * N^S(p(f)), \tag{8}$$

where $1/(1-\delta)$ is a discount term, and the remaining parts constitute the per-period profits. Note that the per-period profits are weighted by $1/2$ because buyers purchase from one of the two sellers. Note that while the price $p = p^{col}$ – and thus also $N^B(p^{col})$ – may be independent of the platform fee under collusion, under seller competition it will be that $p^{com}(f)$ and $N^B(p^{com}(f))$ are both influenced by the platform's fee.

I now show that in case of seller collusion, the platform sets its fee as high as possible to maximize surplus extraction. I then analyze its optimal choice given that sellers compete, before deriving its preferred conduct depending on δ.

6.1 Platform pricing when sellers collude

Suppose the platform prefers that sellers collude. Then, sellers will charge a price of $p^{col} = v$ in each period, which is independent of the fee f, so that the buyer demand $N^B(p^{col})$ will be unaffected by f. Thus, the platform faces the following maximization problem:

$$\max_f \Pi^P(p^{col}, f) = \frac{1}{1-\delta} * \frac{1}{2} * f * N^B(p^{col}) * N^S(p^{col}) \quad \text{s.t. } \delta \geq \delta^*(f) \text{ and } f \leq v - c, \tag{9}$$

where the first constraint is the seller collusion incentive compatibility constraint, and the second constraint is the market coverage assumption (otherwise the platform would generate non-positive profits). Importantly, this implies that the platform's profits are strictly increasing in f. Moreover, since by Lemma 4, larger fees encourage seller collusion and δ approaches zero as f approaches $(v-c)$, the first constraint becomes non-binding if f increases, so the platform sets its fee to extract all surplus. Hence, in case the platform wants to promote collusion, its optimal fee $f^{col} = (v-c)$ guarantees full surplus extraction. The next two Lemmas summarize this finding and provide some comparative statics:

Lemma 5 (Optimal fee – seller collusion)**.** *When the platform wants to promote seller collusion, it sets its fee to $f^{col} = (v-c)$ to extract all surplus.*

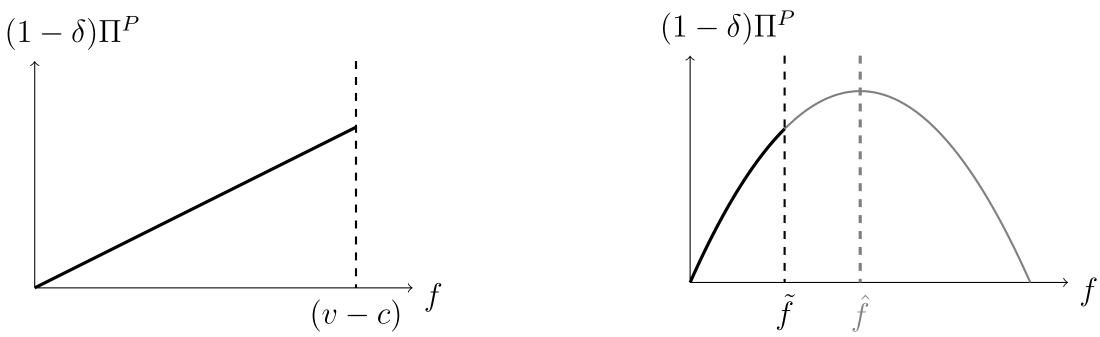
Lemma 6 (Comparative statics of Lemma 5)**.** *When sellers collude, the platform's optimal fee f^{col} is independent of the stand-alone utility a and the product differentiation parameter τ, and increases in the total surplus $(v-c)$.*

A graphical representation of the platform profits depending on f when sellers collude is shown in Figure 2a. I now proceed to the case of seller competition.

6.2 Platform pricing when sellers compete

Suppose now that the platform wants to promote seller competition instead of collusion. As illustrated in Lemma 1 –and in contrast to seller collusion–, if $f \leq v - c - \tau$, the price will then be $p^{com}(f) = c + f + \tau$, which is a function of f, so also $N^B(p^{com}(f))$ will be a function of f (by Corollary 1). As a result, the platform maximizes its discounted profits:

$$\Pi^P(p^{com}(f), f) = \frac{1}{1-\delta} * \frac{1}{2} * f * N^B(p^{com}(f)) * N^S \tag{10}$$



(a) Seller collusion. **(b)** Seller competition.

Figure 2: Platform profits depending on the fee f for $\delta > \bar{\delta}$.

subject to the constraints

$$N^B(p^{com}(f)) = \frac{1}{1-\delta}\left(a + u^{com}(p^{com}(f))\right) \text{ and } N^S = 2, \text{ and } \delta \leq \delta^*(f). \tag{11}$$

As explained above, the fact that $N^S = 2$ follows from the fact that seller demand is inelastic and that under competition, sellers must be ensured to generate a strictly positive profit since products are differentiated. Moreover, as stated by Lemma 4, the critical discount factor $\delta^*(f)$ decreases in f. Hence, in contrast to seller collusion, the platform may be constrained in charging its optimal fee in seller competition in two ways. First, since buyer demand decreases in f, the platform cannot set its fee too high because otherwise, it might lose parts of its users. Second, and more importantly, since a higher fee might induce seller collusion, the platform cannot set its fee too high because otherwise, sellers start to tacitly coordinate on prices. Based on these considerations, the next lemma illustrates the optimal per-unit fee the platform should charge if it wants sellers to compete:

Lemma 7 (Optimal fee – seller competition)**.** *When the platform wants to promote seller competition, there exists a $\overline{\delta} \in (0,1)$ such that the optimal per-unit fee is*

$$f^{com} = \begin{cases} \hat{f} & \text{if } \delta \leq \overline{\delta} \\ \tilde{f} & \text{else} \end{cases} \quad \text{with } \hat{f} = \frac{1}{2}\left(v - c + a - \frac{5\tau}{4}\right) \text{ and } \tilde{f} = v - c - \tau - 4\tau\frac{\delta}{1-\delta}.$$

Lemma 7 shows that if the platform wants to promote competition, the optimal fee f^{com} depends on the level of δ. This dichotomy is driven by the fact that a too-large fee induces seller collusion. In fact, if δ is sufficiently small, sellers are less inclined to collude, and increasing the fee does not have a large impact on their collusion choice, so the optimal fee choice is independent of δ. Therefore, for small δ, the platform is unconstrained in setting its fee if it wants to establish a competitive marketplace acting as a regular monopoly platform.

If δ is large, the platform needs to consider sellers' incentive compatibility constraint when it wants to promote competition via f^{com} because otherwise, it may create incentives to collude. Consequently, for large δ, the optimal fee depends on δ. Therefore, Lemma 7 establishes a trade-off between the optimal fee design and creating collusion incentives for sellers. More precisely, it shows that a platform that wants to induce competition may have to adjust fees downwards. Figure 2b shows this case graphically.

Following this, the next Lemma provides some insights on the comparative statics on whether the platform can set its per-unit fee in an unconstrained manner or not:

Lemma 8 (Comparative statics of Lemma 7)**.** *The threshold value $\overline{\delta}$ for the discount factor of Lemma 7 increases in $(v-c)$, and decreases in a and τ.*

Lemma 8 states that when the stand-alone utility a for buyers who join the platform increases, the platform will more likely be restricted by sellers' collusion incentive compatibility constraint when it wants to promote competition. The reason is that although sellers' collusion incentives do not directly depend on a, they do so indirectly via the fee f: recall from Lemma 4 that collusion is positively related to f, and that when $\delta \leq \bar{\delta}$, f increases in a (see Lemma 7). Together, this implies an indirect link between a and sellers' collusion incentives – intermediated via f. Hence, when a increases, buyers obtain a larger benefit from being active on the platform, and so the number of buyers who join the platform increases both under competition and collusion. Thus, as this leads to more demand, it is optimal for the platform to charge a larger fee, which makes collusion more attractive. To balance this, the platform must therefore set a lower fee to mitigate sellers' collusion incentives.

Likewise, a larger product differentiation parameter τ implies that the platform will be more constrained in setting its optimal fee f^{com} if it wants to promote competition. The reason is that with larger τ, the constrained fee decreases faster than the unconstrained fee. Moreover, since the fee decreases in τ, a larger τ encourages seller collusion. For that reason, when τ increases, the platform must decrease its fee even further to maintain competition.

Conversely, when the available surplus in the marketplace $(v-c)$ increases, the platform is less constrained in setting its fee to promote competition: although larger $(v-c)$ makes collusion more likely, it also increases the fee the platform charges to make sellers indifferent between colluding and competing, counteracting the first effect. Thus, $(v-c)$ can be interpreted as a reduced-form proxy for the strength of indirect network effects.

6.3 Preferred conduct

So far, this section has demonstrated the platform's optimal behavior given the different types of seller conduct. However, given the dynamics above, which is the platform's preferred conduct? And ultimately, which conduct will it promote in its marketplace?

As shown above, depending on δ, the platform can either allow for seller collusion, seller competition in a constrained fashion conditional on sellers' incentives to collude, and seller competition without restrictions. The following proposition presents the main result of this paper. In particular, it shows that the platform's preferred type of seller conduct depends heavily on the discount factor δ:

Proposition 1 (Preferred conduct depending on δ)**.**

(i) For a sufficiently large a or sufficiently low $(v-c)$ such that

$$a - \frac{\tau}{4} + \sqrt{4\tau\left(a - \frac{\tau}{4}\right)} > v - c - \tau,$$

the platform prefers seller collusion for any δ.

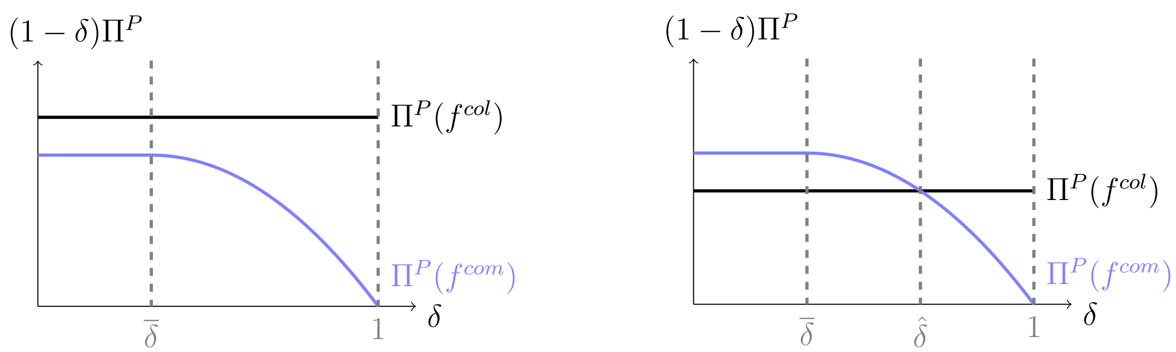
(ii) Additionally, for a sufficiently low a or sufficiently large $(v-c)$ such that

$$v - c - \tau \geq a - \frac{\tau}{4} + \sqrt{4\tau\left(a - \frac{\tau}{4}\right)},$$

the platform prefers seller competition whenever $\delta \leq \hat{\delta}$, and seller collusion whenever $\delta > \hat{\delta}$.

Proposition 1 reports three findings. First, the platform prefers seller collusion whenever the stand-alone utility is sufficiently large, or the surplus available in the market $(v - c)$ is sufficiently low. In fact, a sufficiently large a implies that there is a large number of buyers active in the market when sellers collude. Consequently, if sellers collude, the platform can extract more surplus from the buyer side. Likewise, if $(v - c)$ is sufficiently low, seller collusion is preferred. Even though a decrease in $(v - c)$ implies a decrease in all fees (irrespective of the given conduct), it still holds that $f^{col} > f^{com}$. Therefore, the value of each transaction for the platform is largest when sellers collude. Moreover, the number of buyers may be proportional to $(v - c)$ in case of seller competition when the platform is unconstrained in setting its fee. Finally, since platform profits are always larger in case of seller competition when the platform is unconstrained by sellers' collusion incentives relative to its profits when this constraint is binding, the platform has too few buyers to counterweight the benefit of surplus extraction via the fee. Thus, if $(v - c)$ is small, the platform prefers seller collusion. This relationship is depicted in Figure 3a.

Second, Proposition 1 states that if a is sufficiently small or $(v - c)$ is sufficiently large, then the platform may prefer to establish a competitive marketplace whenever δ is small enough. In fact, if a is small enough, the number of buyers decreases such that the platform's collusion profits are reduced. Similarly, if $(v - c)$ is sufficiently large, so is also buyer demand when sellers compete (while buyer demand is unchanged when sellers collude). Consequently, the platform may want to promote seller competition. However, as for large discount factors sellers might be willing to collude, the platform can promote a competitive marketplace only when δ is low enough. Generally, and as demonstrated in Lemma 7, the platform is able to set its fee unconstrainedly whenever $\delta \leq \bar{\delta}$, but is restricted in doing so whenever $\delta \in (\bar{\delta}, \hat{\delta}]$. Thus, for $\delta > \hat{\delta}$, the platform prefers seller collusion as sellers' collusion incentives are too strong to counteract with the competitive fee.



(a) Platform preferences when a is sufficiently large or $(v - c)$ is sufficiently small.

(b) Platform preferences when a is sufficiently small or $(v - c)$ is sufficiently large.

Figure 3: Preferred conduct by the platform.

Finally, Proposition 1 shows that for a sufficiently low or $(v - c)$ large enough, there is always a parameter range such that platforms that want to induce competition have to adjust fees downwards as $\overline{\delta} \leq \hat{\delta}$. Figure 3b illustrates the last two insights graphically.

Given the mathematical complexity, the intersection value $\hat{\delta}$ at which the platform is indifferent between seller competition and collusion is hard to pin down explicitly. However, I use the Implicit Function Theorem to characterize its comparative statics.

Proposition 2 (Comparative statics of Proposition 1)**.** *The threshold value $\hat{\delta}$ at which the platform is indifferent between seller competition and collusion increases in $(v - c)$, and decreases in a. Moreover, $\hat{\delta}$ increases in τ if $(v - c)$ is sufficiently large or a sufficiently small.*

Proposition 2 states that for a given δ, the platform will tend to promote seller competition instead of seller collusion in case the available surplus $(v-c)$ in its marketplace is rather large (or its stand-alone utility a rather low), and seller collusion when a is sufficiently large (or $(v - c)$ small). This is due to several reasons.

First, note that as a larger discount factor δ makes collusion more likely, the platform would be more constrained in its fee if it wants to promote competition. Thus, an increase in $(v - c)$ leads to an increase in the value of each transaction for the platform. However, as the relative difference between the collusive fee f^{col} and the competitive fee f^{com} decreases as $(v-c)$ increases, the total number of transactions becomes relatively more important than the value of each transaction for the platform. And as there are more buyers when sellers compete (compared to when they collude), it holds that $N^B(p^{com}(f)) > N^B(p^{col})$, which makes collusion relatively less attractive. As a result, when $(v - c)$ increases, the platform will be more likely to encourage seller competition.

Second, to understand why a larger stand-alone utility a fosters seller collusion, recall that a larger a implies more buyers (also when sellers collude), while the value of each transaction remains unchanged. Consequently, as f^{col} and f^{com} remain constant with $f^{col} > f^{com}$ while the numbers of buyers increase in a, the platform can extract more surplus when sellers collude and thereby generate larger profits. Thus, when buyers' stand-alone benefits a are high, the platform is more inclined to foster seller collusion.

Finally, as Proposition 2 shows, the effect of τ on $\hat{\delta}$ is ambiguous. In particular, when a is sufficiently small, the difference in the number of buyers between collusion and competition becomes relatively large, rendering drawing surplus from each transaction relatively more important. Likewise, as a larger τ requires a larger f^{com} when the fee is constrained to make sellers indifferent between colluding and competing, the value per transaction increases in τ. Ultimately, this renders competition relatively more attractive, shifting $\hat{\delta}$ upwards. The converse reasoning holds for a sufficiently large.

7 Alternative fee structures

Thus far, my analysis has focused on platforms imposing per-unit fees. In practice, however, a range of fee structures is available, and the choice of fee design can materially affect market outcomes. In this section, I consider fixed membership fees, profit-sharing fees, and revenue-sharing fees. My results reveal that while many of the qualitative effects identified under per-unit fees carry over to the revenue-sharing setting, the mechanisms operating under fixed membership or profit-sharing fees differ markedly. Yet, for the latter types of fees, platforms always have an incentive to deter entry and thereby limit competition by creating a monopolistic marketplace.

(Fixed) membership fees. In much of the platform economics literature, sellers face a fixed membership fee to access the market. As shown in Appendix 3, when sellers incur such a fee, it enters their profit function as a sunk cost, leaving the collusion condition, Expression (2), independent of it. Thus, the platform is unable to influence sellers' collusion incentives via fee adjustments; the fee instead serves as a device for surplus extraction.

Moreover, if the platform charges only the seller side, it may wish to maximize the pool of sellers so that, in equilibrium, sellers charge a monopoly price to appropriate buyer surplus. Yet, in the case of seller collusion, a platform is, in fact, indifferent between the number of sellers as the platform can always adjust its fee according to the number of sellers. More interestingly, however, in the competitive regime, the platform prefers to have a single seller as this maximizes platform profits. Thus, while membership fees do not directly affect collusion incentives, they can be used to deter entry and thereby create a monopolistic landscape. Finally, since a monopolistic seller is weakly better for the platform, limiting entry via a large membership fee is a weakly dominant action for the platform.[22]

Profit-sharing fees. Appendix 5 extends the analysis to profit-sharing fees, which are particularly relevant for platforms where sellers face negligible marginal production costs (for example, in mobile app stores or online education markets). As with membership fees, Expression (2) does not depend on the profit-sharing fee. In fact, the platform's optimal approach under profit-sharing is to set the fee at 100 percent – thereby capturing the entire seller surplus.

Revenue-sharing fees. Finally, in Appendix 7, I consider revenue-sharing fees. Here, fees scale sellers' marginal costs rather than adding a fixed amount. The per-period seller profits

[22] However, when sellers collude on a price below the monopoly price, one might conjecture that restricting the number of sellers via the fee will be more appealing for the platform as this ensures larger platform profits.

are given by

$$\pi_t^i = (1-f)(p_t^i - c)d_t^i, \tag{12}$$

which leads to a Hotelling equilibrium price of

$$p_t^{com} = \frac{c}{1-f} + \tau \equiv c'(f) + \tau. \tag{13}$$

Thus, the effective unit cost under a revenue-sharing fee is $c/(1-f)$, mirroring the cost increase of $(c+f)$ in the per-unit fee model.[23] In light of this, most of the effects from Sections 4 and 5 hence carry over also to a model of revenue-sharing fees. Nonetheless, this revenue-sharing formulation introduces considerable analytical complexity: the platform's profit function becomes an irrational function of f, and the first-order condition in the competitive regime is a cubic equation. While I can characterize the optimal fee when the platform prefers seller collusion, deriving an analytical solution under competition becomes challenging, rendering the comparative statics intractable. Still, one may obtain sufficient conditions to ensure the concavity of the profit function and thus the existence of a unique competitive revenue-sharing fee.

Finally, I show in Appendix 7 that when platform profits under seller competition exceed those under seller collusion, the behavior with respect to the discount factor δ mirrors that of per-unit fees. Consequently, the resulting profit patterns are similar to those shown in Figures 3a and 3b. While this provides no new analytical insights into how the results might change with respect to the other model parameters, it suggests that the comparative statics of a revenue-sharing model could resemble those of per-unit fees.

8 Policy implications

As the fee's dynamics on seller conduct may heavily depend on the nature of the fee that is imposed by the platform, its policy implications remain very sensitive to the implemented fee type. In the case of profit-sharing or membership fees, policy interventions regarding the size of the fee cannot encourage the platform to self-regulate its marketplace, as these fees do not influence sellers' collusion incentives. Yet, as membership fees do affect seller entry, they are prone to induce monopoly pricing.[24] Therefore, even though the mechanism might differ, the final result may be similar in terms of supra-competitive prices. Thus, if

[23] When $c=0$, revenue and profit are equivalent, making the revenue-sharing model identical to the profit-sharing case. Thus, $c=0$ represents a special case.

[24] Note that the analysis above assumes the sellers' outside option is normalized to zero. If, instead, sellers had a positive outside option, any higher fee would reduce seller profits and could therefore deter entry as well. Hence, this logic also extends to other fee structures.

membership fees are an important feature of platforms, my results may predict a low risk of collusion, but policymakers should pay closer attention to the entry of sellers instead.

Generally, however, for per-unit fees or revenue-sharing fees, the picture might be more nuanced. In fact, by Lemma 4, the platform can influence the sellers' ability to collude via the fee it imposes. Thus, there is a direct link between the level of per-unit fees and collusion: larger fees are more prone to induce collusion, everything else equal. Consequently, capping platform per-unit or revenue-sharing fees (as suggested by Gomes and Mantovani, 2025) might be a remedy to foster seller competition in online marketplaces. Moreover, as Corollary 1 indicates, price caps on platform fees can be beneficial to consumers as they improve the overall participation on the buyer side when sellers compete, and do not decrease welfare in case of seller collusion.

In addition, if one interprets a large surplus $(v - c)$ as an indication of strong network effects, the result of Lemma 5 demonstrates that a platform that wants to promote seller collusion sets a very large fee, which is insensitive to small changes in the network effects. Likewise, as competing sellers tend to incorporate platform fees in their pricing decisions, colluding sellers remain unresponsive to changes in the fee. As such, regulators may perform a SSNIP test directly based on the platform fee to assess whether a collusive agreement exists.[25] Consequently, if pass-through rates of per-unit fees are low, it could indicate seller collusion. Alternatively, one could test for seller collusion based on the comparative statics result of Lemma 6, and check whether the platform adjusts its fee as a response to changes in network effects.

Furthermore, Proposition 1 shows that for a given discount factor, a platform might be willing to promote competition. Conversely, and by Proposition 2, the platform is more likely to promote collusion when network effects are sufficiently weak. This suggests that a more mature platform with a large and established user base is more willing to foster seller collusion than platforms that still face a growing user base (where indirect network effects might play a greater role). As a result, especially well-established platforms might be inclined to promote seller collusion and are thus unable to regulate their marketplace without external interventions.

Closely linked to the previous point, Propositions 1 and 2 suggest that when per-unit fees apply uniformly across product categories, collusion is more likely to inadvertently arise in markets with higher transaction values $(v - c)$, ceteris paribus. This suggests that policymakers should pay closer attention to high-margin sectors, where sellers have stronger incentives to coordinate, and platforms can extract higher rents without deterring sales.

[25] The "SSNIP test" refers to a test of a "small but significant and non-transitory increase in price," which is a common tool in competition policy to assess markets and market power (see, e.g., DoJ, 2004; European Commission, 2024).

Therefore, regulatory scrutiny may be especially warranted in cases where per-unit fees inadvertently encourage collusion in markets with large transaction values.

Finally, as the buyer surplus is proportional to the number of entering buyers, it is clear that buyers enjoy a larger surplus when sellers compete compared to when they collude. Therefore, especially regulators with a focus on consumer surplus should be more concerned about platforms that fail to correct anti-competitive behavior in their marketplaces. Consequently, observing the number of active users on the buyer side on online platforms might serve as an indicator of collusive behavior by sellers tolerated by the platform.

9 Discussion and conclusion

Despite its simplifications, the presented model shows that platforms may find it more profitable to enable collusion than to set fees that foster competition. The analysis offers key insights into platform behavior and informs policy design in digital markets.

Moreover, this paper shows that platforms may have an incentive to foster seller collusion, as colluding sellers extract surplus from buyers, which the platform can capture through its fee. This incentive depends on the fee structure –whether fixed or profit-sharing fees complement per-unit charges– and the strength of indirect network effects. When network effects are weak or the platform is still growing, limiting competition can be more profitable. These findings have key implications for policymakers, emphasizing that network effects can amplify a platform's incentive to encourage collusion. Additionally, they also highlight the importance of distinguishing between established platforms –which may prioritize surplus extraction– and growing platforms –which may focus on expanding their user base. Understanding these dynamics is crucial for designing policies that prevent anti-competitive behavior in digital markets.

However, it is also crucial to recognize that these findings' robustness hinges on the presence of buyer-sided indirect network effects, echoing the previous literature on multi-sided markets (Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006). Future research should explore these mechanisms in more detail, as this model abstracts from many relevant factors. For example, while I analyze different fees separately, platforms often use multiple fees simultaneously: Amazon, for instance, charges revenue-sharing, per-unit, and fixed fees. Yet, little work examines how these interact. Notably, Armstrong and Wright (2007) show that combining fees can lead to multiple equilibria, suggesting a rich area for further study.

In addition, a large body of research studies the implications of platform competition. However, at least for the case of single-homing, my results should remain robust as this can be done by analyzing the role of sellers' outside option, which in my setting is assumed to be homogeneous and normalized to one. As such, I conjecture that the possibility for sellers to enter another platform would dampen, but not fully eliminate, my results. Hence, they remain qualitatively robust to this point.

Finally, the baseline model assumes that sellers face a homogeneous outside option normalized to zero – ensuring that all sellers join as long as they generate non-negative profits, and rendering seller supply inelastic. Although this assumption might appear overly simplistic, it finds some empirical support. In particular, both the Federal Trade Commission's report on multi-sided platform businesses (2014) and the European Commission's report on online platforms (2017) emphasize that buyer-sided network effects are the primary driver of platform value, with seller-side effects playing only a limited role – a finding further corroborated by Scott Morton (2016). Relaxing this assumption to allow for heterogeneous outside options would make seller entry endogenous, so that only those for whom platform participation yields a net benefit would join, potentially amplifying buyer-side externalities and introducing strategic factors like entry thresholds.

In conclusion, my research sheds light on the strategic incentives shaping competition in online marketplaces, showing how platforms may either facilitate seller collusion or, under certain conditions, act to curb it. Whether a platform behaves as a de facto regulator depends on factors such as sellers' discount rates and the strength of network effects.

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Appendix 1 Proofs

Proof of Lemma 1

Proof. Suppose $f \leq v - c - \tau$ and $\delta < \delta^*$, then sellers compete. In that case, each seller $i \in \{0, 1\}$ individually selects a price to maximize her per-period profits. If I denote seller i's competitor by $-i$, then the maximization problem in each period t becomes

$$\max_{p^i} \pi^i = (p - c - f) * \left(\frac{1}{2} + \frac{p^{-i} - p^i}{2\tau}\right), \tag{14}$$

where the first term in the profit function is seller i's profit margin, and the second term is seller i's demand depending on its own price and the price of its competitor.

The first order condition reveals that in a symmetric equilibrium, the price is $p^i = p^{-i} = c + f + \tau \equiv p^{com}(f)$. Moreover, $f \leq v - c - \tau$, so there are no local monopolies. Plugging that price into the profit function reveals that both sellers make a per-interaction profit of $\pi^i = \pi^{-i} = \tau/2 \equiv \pi^{com}$ in each period t. Finally, the (net) surplus for buyers who purchase from seller i is given by

$$u = v - p^i - \frac{\tau}{4} \tag{15}$$

in each period t, so for $p^i = p^{com}(f)$, we obtain $u(f) = v - p^{com}(f) - \tau/4 \equiv u^{com}(f)$. □

Proof of Lemma 2

Proof. If $\delta \geq \delta^*$, then collusion is always subgame-perfect. Also, $f \leq v - c - \tau$, so there are no local monopolies. Hence, colluding sellers collectively select in each period t a price that maximizes their joint profits under the constraint that buyers still purchase, i.e.,

$$\max_{p} \pi = (p^i - c - f) * \frac{1}{2} \quad \text{s.t. } v \geq p. \tag{16}$$

Since seller i's profits increase in p, it selects the highest price possible, so in the symmetric equilibrium, $p^i = p^{-i} = v \equiv p^{col}$, as buyers purchase as long as $p \leq v$. Then, in each period t each seller makes profits equal to $\pi^i = \pi^{-i} = (v - c - f)/2 \equiv \pi^{col}(f)$ per interaction with a buyer, and buyers obtain a per-interaction benefit of $u = -\tau/4 \equiv u^{col}$ per period t. □

Proof of Lemma 3

Proof. Suppose seller $-i$ charges the price p^{col}. If seller i decides to deviate in period t, it selects a price p^i such that

$$\max_{p^i} \pi^i = (p^i - c - f) * \left(\frac{1}{2} + \frac{p^{col} - p^i}{2\tau}\right), \tag{17}$$

which yields $p^i = (v+c+f+\tau)/2 = (p^{col}+p^{com}(f))/2 \equiv p^{dev}(f)$. Therefore, if i charges p^{dev}, it gets a demand equal to

$$d^i = \frac{1}{2} + \frac{v-c-f-\tau}{4\tau} = \frac{1}{2} + \frac{p^{col}-p^{com}(f)}{4\tau} \equiv d^{dev}(f), \tag{18}$$

and generates a per-interaction profit of

$$\pi^i = \frac{1}{2\tau}\left(\frac{v-c-f}{2}+\frac{\tau}{2}\right)^2 = \frac{\left(\pi^{col}(f)+\pi^{com}\right)^2}{2\pi^{com}} \equiv \pi^{dev}(f). \tag{19}$$

Finally, note that $\pi^{dev} \geq \pi^{col} \iff \pi^{col} \geq \pi^{com} \iff v-c-f \geq \tau$, so from a seller's perspective, deviation from the collusive agreement is always profitable whenever colluding is profitable. □

Proof of Lemma 4

Proof. Given sellers' competitive profits and collusion profits from the previous Lemmas, the threshold value for sellers' common discount factor to make collusion subgame-perfect is

$$\delta^*(f) = \frac{\pi^{dev}(f)-\pi^{col}(f)}{\pi^{dev}(f)-\pi^{com}} = \frac{\left(\pi^{col}(f)-\pi^{com}\right)^2}{\left(\pi^{col}(f)+\pi^{com}\right)^2 - 4\left(\pi^{com}\right)^2}, \tag{20}$$

whose derivative with respect to f is proportional to $-(\pi^{col}(f)-\pi^{com})^2$ and therefore negative for any f. □

Proof of Corollary 1

The proof for the entering number of users is just as outlined in the text.

Proof of Lemma 5

Before proving Lemma 5, I first establish another result that will become helpful later in the demonstration.

Lemma 9. *For any discount factor $\delta \in (0,1)$, there exists an $\tilde{f}$ such that $\delta = \delta^*(\tilde{f})$.*

Proof. Since, by Lemma 4,

$$\delta^*(f) = \frac{\left(\frac{v-c-f}{2}-\frac{\tau}{2}\right)^2}{\left(\frac{v-c-f}{2}+\frac{\tau}{2}\right)^2 - 4\left(\frac{\tau}{2}\right)^2}, \tag{21}$$

there exists an $\tilde{f}$ such that for any discount factor δ, it holds

$$\delta = \frac{\left(\frac{v-c-\tilde{f}}{2}-\frac{\tau}{2}\right)^2}{\left(\frac{v-c-\tilde{f}}{2}+\frac{\tau}{2}\right)^2 - 4\left(\frac{\tau}{2}\right)^2}, \tag{22}$$

which can be rearranged to

$$(1-\delta)\left(\frac{v-c-\tilde{f}}{2}-\frac{\tau}{2}\right)^2-(1+\delta)t\frac{v-c-\tilde{f}}{2}-\frac{\tau}{2}+(1+3\delta)\left(\frac{\tau}{2}\right)^2=0. \tag{23}$$

Note that the RHS is quadratic in $(v-c-\tilde{f})/2$, it is also quadratic in $\tilde{f}$, so the equation immediately above holds true for either

$$\frac{v-c-\tilde{f}}{2}=\frac{(1+\delta)t\pm\sqrt{(1+\delta)^2t^2-4(1-\delta)(1+3\delta)\left(\frac{\tau}{2}\right)^2}}{2(1-\delta)} \tag{24}$$

$$\iff \tilde{f}=v-c-\tau-2\tau\frac{\delta}{1-\delta}\pm 2\tau\frac{\delta}{1-\delta}. \tag{25}$$

Therefore,

$$\tilde{f}=\left\{v-c-\tau-4\tau\frac{\delta}{1-\delta};v-c-\tau\right\}<v-c. \tag{26}$$

□

Proving Lemma 5.

Proof. If sellers collude, the price $p^{col}=v$ that buyers face in the marketplace will be independent of f and thus characterized as in Lemma 2, making the number of buyers who join the platform in the second stage independent of f (see Corollary 1). Consequently, the platform's profit maximization problem becomes

$$\begin{aligned}&\max_f \Pi^P(f)=\frac{1}{1-\delta}*f*\frac{1}{2}*\underbrace{\frac{1}{1-\delta}\left(a-\frac{\tau}{4}\right)}_{=N^B(p^{col})}*N^S(p^{col})\\ &s.t.\quad \delta\geq\delta^*(f)\quad\text{and}\quad N^S(p^{col})=\begin{cases}2\text{ if } f\leq v-c\\ 0\text{ else,}\end{cases}\end{aligned} \tag{27}$$

where $\delta^* f$ is defined as in Lemma 4. Note that the objective function is strictly increasing f, so then a maximum exists only if the constraints admit an upper bound on f. Importantly, since the objective function increases in f, only the highest upper bound will be binding. Based on this maximization problem above, I now show that only the second constraint binds in the optimum.

First, note from the second constraint that $f>v-c$ can never be optimal because then $\Pi^P=0$, admitting a potential upper bound on f such that $f=v-c=f^{col}$.

Note further that by Lemma 4 the first constraint decreases in f, so it can only constitute a lower-bound on f. By Lemma 9, it holds that any $f=\tilde{f}<v-c=f^{col}$, so $\Pi^P(f)$ is maximized at $f=f^{col}$. □

Proof of Lemma 6

Proof. Recall that the optimal fee f when sellers collude is $f^{col} = v - c$.

From there, it is straightforward to verify that f^{col} increases in $(v - c)$, and that it is independent of a and t. □

Proof of Lemma 7

I first demonstrate the platform's optimal per-unit fee if it is unconstrained by the seller collusion constraint. Based on that, I continue with the proof of Lemma 7.

Lemma 10. *If the platform is unconstrained by the sellers' collusion incentive compatibility condition the optimal per-unit fee is*

$$\hat{f} = \frac{1}{2}\left(v - c + a - \frac{5\tau}{4}\right)$$

Proof. The first-order condition of the unconstrained maximization problem yields

$$v - c + a - \frac{5\tau}{4} - 2f = 0, \tag{28}$$

which, after solving for f, gives the statement above. □

Proving Lemma 7.

Proof. As $N^S = 2$ and $p^{com}(f) = c + f + \tau$, the platform's maximization problem under seller competition becomes

$$\max_f \Pi^P(f) = \frac{1}{1-\delta} * f * \underbrace{\frac{1}{1-\delta}(a + u^{com}(f))}_{=N^B(p^{com}(f))} \tag{29}$$
$$s.t. \quad \delta \leq \delta^*(f) \quad \text{and} \quad f \leq v - c - t.$$

Note that it must hold that

$$f^{com} = \min\left\{v - c - \tau; \tilde{f}; \hat{f}\right\},$$

where $\tilde{f}$ is defined as in Lemma 9, and

$$\hat{f} = \arg\max_f \Pi^P(f) = \frac{1}{1-\delta} * \frac{1}{2} * f * N^B(p^{com}(f)). \tag{30}$$

is the maximizer of the unconstrained optimization problem.

Notice first that by Lemma 9, $\tilde{f} \leq v - c - \tau$, so $f^{com} = \min\left\{\tilde{f}; \hat{f}\right\}$. Note further that by Lemma 10

$$\hat{f} = \frac{1}{2}\left(v - c + a - \frac{5\tau}{4}\right). \tag{31}$$

Based on this, the platform is unconstrained by sellers' incentive compatibility constraint in setting its fee when sellers compete if $\hat{f} \leq \tilde{f}$, and constrained otherwise.

Thus, the platform can maintain seller competition with an unconstrained fee if $\hat{f} \leq \tilde{f}$, or

$$\frac{1}{2}\left(v - c + a - \frac{5\tau}{4}\right) \leq v - c - \tau - 4\tau\frac{\delta}{1-\delta} \tag{32}$$

$$\iff \quad \frac{1}{2}\left(a - \frac{\tau}{4}\right) \leq \frac{v-c-\tau}{2} - 4\tau\frac{\delta}{1-\delta} \tag{33}$$

which can be rearranged to

$$4\tau\frac{\delta}{1-\delta} \leq \frac{1}{2}\left(v - c - \tau - \left(a - \frac{\tau}{4}\right)\right) \tag{34}$$

$$\iff \quad \delta \geq (1-\delta)\frac{1}{8\tau}\left(v - c - \tau - \left(a - \frac{\tau}{4}\right)\right) \tag{35}$$

$$\iff \quad \delta\left(1 + \frac{1}{8\tau}\left(v - c - \tau - \left(a - \frac{\tau}{4}\right)\right)\right) \leq \frac{1}{8\tau}\left(v - c - \tau - \left(a - \frac{\tau}{4}\right)\right) \tag{36}$$

$$\iff \quad \delta = \frac{v - c - \tau - \left(a - \frac{\tau}{4}\right)}{v - c - \tau - \left(a - \frac{\tau}{4}\right) + 8\tau} \equiv \bar{\delta}. \tag{37}$$

This completes the proof. □

Proof of Lemma 8

Proof. Recall from the Proof of Lemma 7 that

$$\bar{\delta} = \frac{v - c - \tau - \left(a - \frac{\tau}{4}\right)}{v - c - \tau - \left(a - \frac{\tau}{4}\right) + 8\tau}. \tag{38}$$

From there it is easy to verify that

$$\frac{d\bar{\delta}}{da} = -\frac{8\tau}{\left(v - c - \tau - \left(a - \frac{\tau}{4}\right) + 8\tau\right)^2} < 0, \tag{39}$$

$$\frac{d\bar{\delta}}{d(v-c)} = \frac{8\tau}{\left(v - c - \tau - \left(a - \frac{\tau}{4}\right) + 8\tau\right)^2} > 0, \text{ and} \tag{40}$$

$$\frac{d\bar{\delta}}{d\tau} = -\frac{8\left(v - c - \tau - \left(a - \frac{\tau}{4}\right)\right) + 6\tau}{\left(v - c - \tau - \left(a - \frac{\tau}{4}\right) + 8\tau\right)^2} < 0. \tag{41}$$

□

Proof of Proposition 1

To prove this Proposition, I first develop three Lemmas as they will be useful for the demonstration of Proposition 1.

Lemma 11. *Based on Lemma 5, in case of seller collusion the platform makes profits equal to*

$$\Pi^P(p^{col}, f^{col}) = \frac{1}{(1-\delta)^2} * (v - c)\left(a - \frac{\tau}{4}\right).$$

Proof. By Lemma 5, it hold $f^{col} = v - c$. Likewise, by Corollary 1, it holds $N^B(p^{col}) = 1/(1-\delta)(a - \tau/4)$, so total profits are equal to

$$\Pi^P(v-c) = \frac{1}{(1-\delta)^2} * (v-c)\left(a - \frac{\tau}{4}\right). \tag{42}$$

□

Lemma 12. *Based on Lemma 7, in case of seller competition with $\delta \leq \bar{\delta}$ the platform profits are equal to*

$$\Pi^P(p^{com}(f^{com})) = \frac{1}{(1-\delta)^2} * \frac{1}{4} * \left(v - c + a - \frac{5\tau}{4}\right)^2.$$

Proof. By Lemma 7, it holds that $f^{com} = v - c + a - 5\tau/4$ for $\delta \leq \bar{\delta}$. Likewise, by Corollary 1, it holds $N^B(p^{com}(f)) = 1/(1-\delta)(v - c - f^{com} - 5\tau/4)/2$, so

$$N^B(f^{com}) = \frac{1}{1-\delta}\frac{1}{4}\left(v - c + a - \frac{5\tau}{4}\right)^2 \tag{43}$$

if $\delta \leq \bar{\delta}$. Consequently, total profits are equal to

$$\Pi^P(p^{com}(f^{com})) = \frac{1}{(1-\delta)^2} * \frac{1}{4}\left(v - c + a - \frac{5\tau}{4}\right)^2.$$

if $\delta \leq \bar{\delta}$. This completes the proof. □

Lemma 13. *Based on Lemma 7, in case of seller competition with $\delta \geq \bar{\delta}$ the platform profits are equal to*

$$\Pi^P(p^{com}(f^{com})) = \frac{1}{(1-\delta)^2} * \left(v - c - \tau - 4\tau\frac{\delta}{1-\delta}\right)\left(a - \frac{\tau}{4} + 4\tau\frac{\delta}{1-\delta}\right).$$

Proof. By Lemma 7, it hold $f^{com} = v - c - \tau - 4\tau\delta/(1-\delta)$ for $\delta \geq \bar{\delta}$. Likewise, by Corollary 1, it holds $N^B(p^{com}) = 1/(1-\delta)(v - c - f^{com} + a - 5\tau/4)/2)$, so total profits are equal to

$$\Pi^P(p^{com}(f^{com})) = \frac{1}{(1-\delta)^2} * \left(v - c - \tau - 4\tau\frac{\delta}{1-\delta}\right)\left(a - \frac{\tau}{4} + 2\tau\frac{\delta}{1-\delta}\right).$$

if $\delta \geq \bar{\delta}$. □

Proving Proposition 1. Based on the three Lemmas above, I now proceed with the demonstration of Proposition 1.

Proof. To show the optimal platform fee depending on δ, we need to compare the platform profits in all three contingencies. The profits from seller collusion and competition are given, respectively, by the Lemmas 11, 12, and 13.

First, notice that when $\Pi^P(f^{col}) \geq \Pi^P(f^{com})$ for $\delta \leq \bar{\delta}$, then the platform will always prefer collusion for all other levels of $\delta \in (0,1)$ as well. This is the case whenever

$$\frac{1}{(1-\delta)^2} * (v-c)\left(a - \frac{\tau}{4}\right) \geq \frac{1}{(1-\delta)^2} * \frac{1}{4}\left(v - c - +a - \frac{5\tau}{4}\right)^2 \tag{44}$$

$$\iff \left(v - c - \tau - \left(a - \frac{\tau}{4}\right)\right)^2 - 4\tau\left(a - \frac{\tau}{4}\right) \leq 0, \tag{45}$$

which is quadratic in a, $(v-c)$, and t. Moreover, since its discriminant is

$$16\tau\left(a - \frac{\tau}{8}\right), \tag{46}$$

there exist two real roots:

$$v - c - \tau - \left(a - \frac{\tau}{4}\right) = \pm\sqrt{4\tau\left(a - \frac{\tau}{4}\right)}. \tag{47}$$

However, notice that $v - c - \tau) > a - \tau/4$ by Expression (4), it holds that only one root is admissable, such that

$$v - c - \tau - \left(a - \frac{\tau}{4}\right) = \sqrt{4\tau\left(a - \frac{\tau}{4}\right)} \quad \Longrightarrow \quad v - c - \tau \geq \left(a - \frac{\tau}{4}\right) + \sqrt{4\tau\left(a - \frac{\tau}{4}\right)}, \tag{48}$$

so a must be either low enough, or $(v-c)$ sufficiently large for this statement to hold.

Second, notice that when $\Pi^P(f^{col}) < \Pi^P(f^{com})$ for $\delta \leq \bar{\delta}$, then by the Intermediate Value Theorem and the monotonicity of $\Pi^P(f^{com})$ there exists a unique δ such that

$$\hat{\delta} = \left\{\delta : \Pi^P(f^{col}) = \Pi^P(f^{com}) \text{ for } \delta \geq \bar{\delta}\right\}. \tag{49}$$

I now demonstrate that $(1-\delta)^2\Pi^P(f^{com})$ is indeed monotonically decreasing in δ. To see this, notice first that from the Lemmas 12 and 13,

$$2(1-\delta)^2\Pi^P(f^{col}) = \begin{cases} \frac{1}{4}\left(v - c + a - \frac{5\tau}{4}\right)^2 & \text{if } \delta \leq \delta^* \\ \left(v - c - \tau - 4\tau\frac{\delta}{1-\delta}\right)\left(a - \frac{\tau}{4} + 4\tau\frac{\delta}{1-\delta}\right) & \text{else}, \end{cases} \tag{50}$$

so

$$\frac{d2(1-\delta)^2\Pi^P(f^{col})}{d\delta} = 0 \quad \text{if } \delta \leq \delta^*. \tag{51}$$

Moreover, recall that for $\delta > \delta^*$, f^{com} is determined by the corner solution $\tilde{f} < \hat{f}$, so

$$\frac{d\,2(1-\delta)\Pi^P(f^{col})}{df} > 0 \tag{52}$$

at $f = \tilde{f}$. In addition, since $\tilde{f} = v - c - \tau - 4\tau\delta/(1-\delta)$,

$$\frac{d\tilde{f}}{d\delta} = -4\tau\frac{1}{(1-\delta)^2} < 0. \tag{53}$$

Finally, notice that δ affects $2(1-\delta)^2\Pi^P(\tilde{f})$ only indirectly via $\tilde{f}$, so

$$\frac{d\,2(1-\delta)^2\Pi^P(f^{com})}{d\delta} = \underbrace{\left.\frac{d\,2(1-\delta)^2\Pi^P(f^{com})}{df}\right|_{f=\tilde{f}}}_{>0} * \underbrace{\frac{d\tilde{f}}{d\delta}}_{<0} < 0. \tag{54}$$

Moreover, by Lemma 11, it holds that

$$2(1-\delta)^2\Pi^P(f^{col}) = (v-c)\left(a - \frac{\tau}{4}\right), \tag{55}$$

so

$$\frac{d\,2(1-\delta)^2\Pi^P(f^{col})}{d\delta} = 0. \tag{56}$$

Therefore, if $\Pi^P(f^{com}) > \Pi^P(f^{col})$ for $\delta \leq \delta^*$, it must hold that $\Pi^P(f^{com})$ crosses $\Pi^P(f^{col})$ at $\delta \equiv \hat{\delta} \in [\bar{\delta}, 1)$. As a result, for $\delta \equiv \hat{\delta} \in [\bar{\delta}, 1)$, the platform prefers seller collusion. □

Proof of Proposition 2

Proof. I prove this Proposition by using the Implicit Function Theorem. Throughout this proof, I will assume that $\overline{\delta} > 0$, because else $\Pi^P(f^{col}) > \Pi^P(f^{com})$ for all $\delta \in (0,1)$ and so $\hat{\delta} \in (0,1)$ does not exist.

To start, note first that for $\hat{\delta}$, it holds that $\Pi^P(\tilde{f}) = \Pi^P(f^{col})$, so I can denote a function F such that

$$F \equiv 2(1-\delta)^2 \left(\Pi^P(\tilde{f}(\hat{\delta})) - \Pi^P(f^{col})\right) \tag{57}$$

$$= \left(v - c - \tau - 4\tau\frac{\hat{\delta}}{1-\hat{\delta}}\right)\left(a - \frac{\tau}{4} + 4\tau\frac{\hat{\delta}}{1-\hat{\delta}}\right) - (v-c)\left(a - \frac{\tau}{8}\right) = 0. \tag{58}$$

Note further that $\hat{\delta}/(1-\hat{\delta})$ is strictly increasing in $\hat{\delta}$. Therefore, if I denote $y \equiv \hat{\delta}/(1-\hat{\delta})$, I can perform the comparative statics equivalently for y instead of $\hat{\delta}$. Thus for $x = \{a, (v-c), \tau\}$ and $y = \hat{\delta}/(1-\hat{\delta})$, $F = F(x,y)$, so by the Implicit Function Theorem:

$$\frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y} \quad \text{for} \quad F(x,y) = 0. \tag{59}$$

I divide this proof in four steps.

Step 1. Change of F with respect to y.
Before analyzing the comparative statics, note that each term in $F(x,y)$ is positive. Note, too, that

$$\frac{\partial F}{\partial y} = -4\tau\left(a - \frac{\tau}{4} + 4\tau y\right) + 4\tau\left(v - c - \tau - 4\tau y\right) \tag{60}$$

$$= -4\tau\left(v - c - \tau - \left(a - \frac{\tau}{4}\right) - 8\tau y\right), \tag{61}$$

which is negative for any $\overline{\delta} > 0$. To see this, take $\delta = \overline{\delta}$, and so

$$y|_{\delta=\overline{\delta}} = \frac{v - c - \tau - \left(a - \frac{\tau}{4}\right)}{v - c - \tau - \left(a - \frac{\tau}{4}\right) + 8\tau} = \frac{v - c - \tau - \left(a - \frac{\tau}{4}\right)}{8\tau}, \tag{62}$$

which yields

$$\left.\frac{dF}{dy}\right|_{\delta=\overline{\delta}} = v - c - \tau - \left(a - \frac{\tau}{4}\right) - 8\tau\frac{v - c - \tau - \left(a - \frac{\tau}{4}\right)}{8\tau} = 0. \tag{63}$$

Thus, for any $\delta > \overline{\delta}$, this expression becomes negative.

Step 2. Change with respect to $(v-c)$.
Following Step 1, note first that

$$\frac{\partial F}{\partial(v-c)} = a - \frac{\tau}{4} + 4\tau y - \left(a - \frac{\tau}{4}\right) = 4\tau y \quad > 0, \tag{64}$$

so
$$\frac{dy}{d(v-c)} = \frac{\partial F/\partial(v-c)}{\partial F/\partial y} = -\frac{4\tau y}{\partial F/\partial y} \quad > \ 0 \quad \text{for} \quad \overline{\delta} > 0, \tag{65}$$
so $\hat{\delta}$ is strictly increasing in $(v-c)$ for $\overline{\delta} > 0$.

Step 3. Change with respect to a.

Second, note that
$$\frac{\partial F}{\partial a} = v - c - \tau - 4\tau y - (v-c) = -\tau(1+4y) \quad < \ 0, \tag{66}$$
yielding
$$\frac{dy}{da} = \frac{\partial F/\partial a}{\partial F/\partial y} = -\frac{-\tau(1+4y)}{\partial F/\partial y} \quad < \ 0 \quad \text{for} \quad \overline{\delta} > 0. \tag{67}$$
Hence, $\hat{\delta}$ is strictly decreasing in a for $\overline{\delta} > 0$.

Step 4. Change with respect to τ.

Lastly, it holds that
$$\frac{\partial F}{\partial \tau} = -(1+4y)\left(a - \frac{\tau}{4} + 4\tau y\right) + \left(-\frac{1}{4} + 4y\right)(v - c - \tau - 4\tau y) + \frac{1}{4}(v-c), \tag{68}$$
which can be rearranged to
$$4y\left(v - c - \tau + a - \frac{\tau}{4}\right) - a + \tau\left(\frac{1}{2} - 3y\right). \tag{69}$$
Observe that $dy/d\tau > 0$ if $\partial F/\partial \tau > 0$, so $\hat{\delta}$ increases in τ if
$$4y\left(v - c - \tau + a - \frac{\tau}{4}\right) + \frac{\tau}{4}(1+4y) - \left(a - \frac{\tau}{4}\right) - 4\tau y > 0 \tag{70}$$
$$\iff \quad 4y(v - c - \tau) + \frac{\tau}{4} - 3\tau y > (1 - 4\tau y)\left(a - \frac{\tau}{4}\right), \tag{71}$$
which can be rearranged to
$$\frac{4y}{1-4y}(v - c - \tau) + \frac{\tau/4 - 3\tau y}{1-4y} > a - \frac{\tau}{4} \tag{72}$$
if $1 > 4y$. Consequently, if a is sufficiently small such that $\partial F/\partial \tau > 0$, then $\hat{\delta}$ is strictly increasing in τ. □

Appendix 2 Motivation for per-unit fees

In this appendix, I argue that per-unit fees are a key feature of e-commerce, shaping platform revenue models across industries. Online marketplaces like Amazon and eBay rely on per-unit fees: Amazon's Individual selling plan includes a $\$0.99$ fee per item, while eBay charges a per-order final value fee. Food delivery platforms, such as Uber Eats and DoorDash, also use per-unit fees alongside revenue-sharing charges.

Beyond retail, per-unit fees are common in event ticketing and print-on-demand. Platforms like Universe and Brown Paper Tickets charge per-ticket fees, allowing event organizers to scale without high upfront costs. Similarly, the print-on-demand platforms Printful and Printify use per-unit fees to cover production and fulfillment costs. They argue these fees benefit the platform and sellers alike, creating a scalable and performance-driven marketplace to accommodate a great number of users.

These examples, summarized in Table 1 with their sources in Appendix 11, illustrate the widespread use of per-unit fees in two-sided markets. While not exhaustive, they highlight the role of these fees in balancing platform profitability and user participation.

Table 1: Examples of platforms using per-unit fees

Field of business	**Platform and type of per-unit fees**
Online marketplaces	Amazon (individual selling plan and fulfillment fees), eBay (final value fee)
Food delivery	Uber Eats (per-unit fee), DoorDash (merchant fee)
Event ticketing	Universe (selling fee for each ticket), Brown Paper Tickets (fee per ticket)
Print-on-demand	Printful (branding options for each unit sold), Printify (paid per item sold)

Appendix 3 Membership fees

In this section, I modify the analysis to incorporate membership fees. Membership fees are typically charged as a fixed amount of money that users pay to the platform on a periodical basis, irrespective of their size, sales, or profits. In fact, Table 2 shows that membership fees are used by several platforms across different business fields. The sources for Table 2 can be found at the end of this section.

I first adjust the baseline model of Section 3 analyzing a game with membership fees. Note that even though parts of the model change, the timeline of the game remains unchanged. Moreover, I show in the analysis that the platform cannot influence sellers' collusion incentives since membership fees do not affect the critical discount factor required for sellers to sustain tacit collusion.

A2.1 Model with membership fees

In order to feature membership fees, I need to make a few minor adjustments. In the first stage, the platform sets the membership fee f. As before, the platform only charges the seller side, and it has no marginal cost for serving its users.

In the second stage, there are infinitely many sellers and buyers who can join the platform or not. Their valuations are given, respectively, by

$$v^S(f) = \sum_{t=0}^{\infty} \delta^t \pi_t(f) * N^B \quad \text{and} \quad v^B = \sum_{t=0}^{\infty} \delta^t \left(a + u_t * \frac{1}{2} N^S(f) \right), \tag{73}$$

where periods are denoted by $t = \{1, 2, ...\}$. Except for the fee f, all model primitives and the interpretation of the other objects remain as in Section 3.

As before, the third stage involves an infinitely repeated static game where sellers can decide either to collude or to compete. Therefore, the seller incentive compatibility constraint for collusion becomes

$$\delta \geq \frac{\pi_t^{dev} - \pi_t^{col}}{\pi_t^{dev} - \pi_t^{com}}. \tag{74}$$

Just as before, I solve the game by looking for subgame-perfect equilibria.

Table 2: Examples of platforms using membership fees

Business field	**Platform and type of membership fees**
Online marketplaces	Alibaba (subscriptions), Amazon (professional selling Plan), eBay (subscriptions), Etsy (Etsy Plus)
Food related	Nem TakeAway (subscriptions), Too Good To Go (platform fee)
Recruiting	LinkedIn (subscriptions), StepStone (subscriptions)
Digital payments	Square (subscriptions)

A2.2 Seller behavior and collusion incentives with membership fees

Because the platform's membership fee and user entry decisions are set before the third stage, they can be treated as exogenous. The next three lemmas characterize the competitive and collusive equilibria in the repeated game and derive the optimal deviation strategy for a seller breaking a collusive agreement. Proofs for all results in this Section appear in Appendix 4.

Lemma 14 (Competitive seller pricing (membership fees)). *If $\delta < \delta^*$, there is a unique symmetric competitive equilibrium such that in each period t,*

$$p^{com} = c + \tau, \quad \pi^{com}(f) = \frac{\tau}{2} - f, \quad \textit{and} \quad u^{com} = v - p^{com} - \frac{\tau}{4}$$

if $f \leq \tau/2$. Moreover, if $f > \tau/2$ and $\delta < \delta^*$, there is a unique monopolistic equilibrium such that there is a single seller who in each period t charges p^{com} such that

$$p^{com} = v, \quad \pi^{com}(f) = v - c - f, \quad \textit{and} \quad u^{com} = -\frac{\tau}{4}.$$

Finally, if $f > v - c$, then buyers do not purchase.

Lemma 15 (Collusive seller pricing (membership fees)). *If $\delta > \delta^*$ and $f \leq (v-c)/2$, there is a unique symmetric collusive equilibrium such that in each period t,*

$$p^{col} = v, \; \pi^{col}(f) = \frac{v-c}{2} - f, \; \textit{and} \; u^{col} = -\frac{\tau}{4}.$$

Additionally, if $\delta > \delta^*$ and $f \geq (v-c)/2$, there is a unique monopolistic equilibrium such that there is a single seller who in each period t charges p^{com} such that

$$p^{col} = v, \quad \pi^{col}(f) = v - c - f, \quad \textit{and} \quad u^{col} = -\frac{\tau}{4}.$$

Finally, if $f > v - c$, then buyers do not purchase.

Lemma 16 (Deviating seller pricing (membership fees)). *If the platform charges a membership fee $f \leq (v-c)/2$ and a seller deviates from the collusive agreement in any given period t, the deviating seller charges a price p_t^{dev} and gets a demand d_t^{dev} such that*

$$p_t^{def} = \frac{p^{col} + p^{com}}{2}, \quad d_t^{dev} = \frac{1}{2} + \frac{p^{col} - p^{com}}{4\tau} \quad \textit{and} \quad \pi_t^{com}(f) = \frac{1}{2\pi}\left(\frac{v-c}{2} + \frac{\tau}{2}\right)^2 - f.$$

Importantly, a deviation is only possible if more than one seller is active, which requires $f \leq (v-c)/2$ (see Lemma 16). More interestingly, and in contrast to per-unit fees, seller profits under membership fees are additive in f. As the next lemma shows, this additivity eliminates the platform's ability to influence sellers' collusion incentives.

Lemma 17 (Collusion incentives (membership fees)). *The threshold value for sellers' common discount factor $\delta^*(f)$ for future profits to make collusion subgame-perfect is independent of the platform's membership fee f.*

Because the membership fee enters seller profits as an additive constant, it drops out of the incentive compatibility constraint for collusion. Figure 4 illustrates that the critical discount factor remains unchanged as the membership fee varies.

Since the platform's membership fee f has no effect on the critical discount factor δ^*, it cannot be used to influence seller conduct. While the platform could, in principle, set f above sellers' per-interaction profits net of the fee, doing so would drive all sellers out of the market and is therefore never optimal. Any such fee is strictly dominated by one that allows at least some seller participation. As a result, the platform sets $f = \tau/2$ under competition and $f = (v-c)/2$ under collusion.

A2.3 User entry with membership fees

I now turn to the user entry decisions. It follows directly that when sellers compete, the number of active sellers is $N^S(f) = 2$ if $f \leq \tau/2$ and $N^S(f) = 1$ if $f > \tau/2$. Likewise, when sellers collude, $N^S(f) = 2$ if $f \leq (v-c)/2$ and $N^S(f) = 1$ if $f > (v-c)/2$. Since no buyer purchases at $f > v-c$ (see Lemma 14), I restrict the analysis to cases where $f \leq v-c$.

Next, consider buyer entry. When sellers compete,

$$N^B(p^{com}) = \begin{cases} \frac{1}{1-\delta}\left(v-c+a-\frac{5\tau}{4}\right) & \text{if } f \leq \frac{\tau}{2}, \\ \frac{1}{1-\delta}\left(a-\frac{\tau}{4}\right) & \text{if } f > \frac{\tau}{2}. \end{cases} \tag{75}$$

When sellers collude,

$$N^B(p^{col}) = \frac{1}{1-\delta}\left(a-\frac{\tau}{4}\right). \tag{76}$$

A2.4 Platform governance and preferred conduct with membership fees

In this part, I establish that restricting the number of sellers is a weakly dominant action for the platform. To do so, I first study the platform's optimal fee choice when sellers collude, and compare it later to the case when sellers compete.

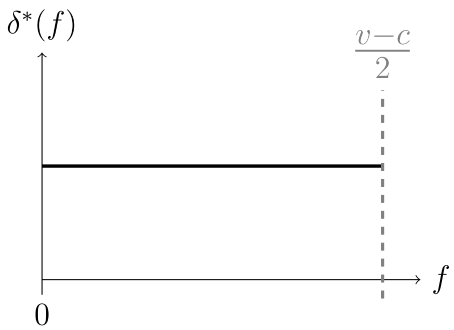

Figure 4: Collusion incentives depending on the fee f with membership fees.

I now examine the platform's optimal strategy when sellers collude. If two sellers enter ($N^S(f) = 2$), the platform maximizes

$$\max_f \Pi^P = \frac{1}{1-\delta} * 2f \quad \text{s.t. } f \leq \frac{v-c}{2}. \tag{77}$$

Since Π^P is strictly increasing in f, the platform sets $f_2^{col} = (v-c)/2$, yielding profits of

$$\Pi^P(f_2^{col}) = \frac{v-c}{1-\delta}. \tag{78}$$

Alternatively, if only one seller enters ($N^S(f) = 1$), the platform solves

$$\max_f \Pi^P = \frac{1}{1-\delta} * f \quad \text{s.t. } f \leq v-c. \tag{79}$$

Again, since Π^P is increasing in f, the platform sets $f_1^{col} = v-c$, yielding the same profits as with two sellers. As a result, the platform is indifferent between having one or two sellers when sellers collude.

Now, consider the case where sellers compete. If the platform accommodates two sellers ($N^S(f) = 2$), it maximizes

$$\max_f \Pi^P = \frac{1}{1-\delta} * f \quad \text{s.t. } f \leq \frac{\tau}{2}, \tag{80}$$

which leads to $f_2^{com} = \tau/2$ and

$$\Pi^P(f_2^{com}) = \frac{1}{1-\delta} * \frac{\tau}{2}. \tag{81}$$

If, instead, the platform allows only one seller to enter ($N^S(f) = 1$), it solves

$$\max_f \Pi^P = \frac{1}{1-\delta} * f \quad \text{s.t. } f \leq v-c. \tag{82}$$

so that $f_1^{com} = f_1^{col} = v-c$ and

$$\Pi^P(f_1^{com}) = \Pi^P(f_1^{col}) = \frac{v-c}{1-\delta}. \tag{83}$$

Since $v > c + \tau$, it follows that $\Pi^P(f_1^{com}) > \Pi^P(f_2^{com})$, so the platform strictly prefers to accommodate only one seller.

Lemma 18 (Preferred conduct (membership fees)). *Under membership fees, restricting seller entry to ensure a monopolistic marketplace is a weakly dominant strategy for the platform.*

Thus, when sellers collude, the platform is indifferent between accommodating one or two sellers, as it can extract the same profits in both cases. In contrast, when sellers compete, the platform strictly prefers limiting entry to a single seller, as this allows it to charge a higher fee and generate greater profits. As a result, the platform has a weakly dominant strategy of restricting seller entry to create a monopolistic marketplace.

A2.5 Sources of Table 2

Online marketplaces

- **Alibaba.** Sellers on Alibaba can choose between two types of monthly subscriptions to gain access to the platform. Source: https://seller.alibaba.com/us/pricing?spm=a272f.28963383.2130016610.2.3d2d58dceMdMyc. Visited on January 14, 2025.

- **Amazon.** Amazon provides sellers the possibility to use a 'professional selling plan', where sellers pay a fixed amount of money each month. Source: https://sell.amazon.com/pricing. Visited on January 14, 2025.

- **eBay.** There are multiple types of subscriptions for sellers on eBay. Source: https://www.ebay.com/sellercenter/payments-and-fees/subscriptions-and-fees. Visited on January 14, 2025.

- **Etsy**. The online marketplace Etsy offers sellers the possibility to use 'Etsy Plus', which involves a monthly subscription. Source: https://www.etsy.com/legal/fees/#fee-types. Visited on January 14, 2025.

Food related

- **Nem TakeAway.** The Danish food delivery platform Nem TakeAway offers several monthly subscription plans for restaurants. Source: https://www.nemtakeaway.dk/en/price/. Visited on January 14, 2025.

- **Too Good To Go.** The platform charges sellers an annual 'platform fee' to be active on it. Source: https://www.toogoodtogo.com/en-us/terms-and-conditions-marketplace. Visited on January 14, 2025.

Recruiting

- **LinkedIn.** The recruiting portal LinkedIn offers a variety of different subscriptions on a monthly basis for companies to advertise open vacancies. Source: https://www.linkedin.com/help/linkedin/answer/a545596?src=direct%2Fnone&veh=direct%2Fnone. Visited on January 14, 2025.

- **StepStone.** Similar to LinkedIn, StepStone has several subscriptions to post jobs. Source: https://www.stepstone.be/wp-content/uploads/2019/02/Pricelist-document_01_2019_EN.pdf. Visited on January 14, 2025.

Digital payments

- **Square.** Apart from royalties, Square offers participating businesses various monthly subscription plans. Source: https://squareup.com/us/en/pricing. Visited on January 14, 2025.

Appendix 4 Proofs for Appendix 3

Proof of Lemma 14

The first part for the case of $f \leq \tau/2$ of the proof is analogous to the proof of Lemma 1.

For the second part, note that with $f \in (\tau/2, v - c]$, only one seller is active. As a result, the active sellers select the profit-maximizing price v and get all the demand on the Hotelling line, yielding the per-interaction profits and purchase utility as stated in the Lemma.

Finally, in the last case when $f > v - c$, sellers charge a price larger than v, so nobody buys. This completes the proof.

Proof of Lemma 15

As before, the first part for the case of $f \leq \tau/2$ of the proof is analogous to the proof of Lemma 2.

The second and third parts are analogous to the second and third parts of the proof of Lemma 14.

Finally, in the last case when $f > v - c$, sellers charge a price larger than v, so nobody buys. This completes the proof.

Proof of Lemma 16

This proof is analogous to the proof of Lemma 3.

Proof of Lemma 17

Proof. After modifying sellers' competitive profits and collusion profits from the previous Lemmas to with the unit costs c, the threshold value for sellers' common discount factor to make collusion subgame-perfect is

$$\delta^*(f) = \frac{\pi^{dev}(f) - \pi^{col}(f)}{\pi^{dev}(f) - \pi^{com}(f)} = \frac{\left(\pi^{col}(f) - \pi^{com}(f)\right)^2}{\left(\pi^{col}(f) + \pi^{com}(f)\right)^2 - 4\left(\pi^{com}(f)\right)^2}, \tag{84}$$

whose derivative with respect to f is equal to zero and therefore independent of any f. □

Proof of Lemma 18

Proof. The proof is straightforward from the analysis. □

Appendix 5 Profit-sharing fees

In this section, I modify the analysis to incorporate profit-sharing fees. If the platform employs profit-sharing fees, it taxes a percentage term from each seller's profits. Therefore, profit-sharing fees typically require that the platform has perfect information about sellers' profits. This might be applicable to online app stores, video game platforms, e-learning platforms, or digital payment platforms like PayPal or Square. For online app stores or video game platforms, for instance, developers might face a fixed cost or an up-front investment to create an app, but typically have negligible to no marginal costs in serving its users. Likewise, teachers on e-learning platforms usually create short movies dedicated to special topics that they offer in packages to multiple e-learners. For further examples, see Table 3.

To analyze a game with profit-sharing fees, I first adjust the baseline model of Section 3. Note that even though parts of the model change, the timeline of the game remains unchanged. Moreover, I show in the analysis that the platform cannot influence sellers' collusion incentives since profit-sharing dows not affect the critical discount factor required for sellers to sustain tacit collusion. As such, the results in this Section mimic those from Appendix 3 with membership fees.

A4.1 Model with profit-sharing fees

In order for my model to feature profit-sharing fees, I need to make a few minor adjustments. In the first stage, the platform sets the profit-sharing fee $f \geq 0$. As before, the platform only charges the seller side, and it has no marginal cost for serving its users.

In the second stage, there are infinitely many sellers and buyers who can join the platform or not. Their valuations are given, respectively, by

$$v^S(f) = \sum_{t=0}^{\infty} \delta^t \pi_t(f) * N^B \quad \text{and} \quad v^B = \sum_{t=0}^{\infty} \delta^t \left(a + \frac{1}{2} u_t * N^S(f) \right), \tag{85}$$

where periods are denoted by $t = \{1, 2, ...\}$. Except for the fee f, all model primitives and the interpretation of the other objects remain as in Section 3.

As before, the third stage involves an infinitely repeated static game where sellers can decide either to collude or to compete. Therefore, the seller incentive compatibility constraint

Table 3: Examples of platforms most likely to effectively use profit-sharing fees

Business field	Platform and type of profit-sharing fees
App stores	Google Play Store, Apple App Store, Amazon App Store
Video games	Steam, Epic Games Store
E-learning	Coursera, Udemy, LearnWorlds
Digital payments	PayPal, Square

for collusion becomes

$$\delta \geq \frac{\pi_t^{dev} - \pi_t^{col}}{\pi_t^{dev} - \pi_t^{com}}. \tag{86}$$

As before, I solve the game by looking for subgame-perfect equilibria.

A4.2 Seller behavior and collusion incentives with profit-sharing fees

As the platform's fee and user entry decisions are predetermined in the third stage, I can treat them as exogenous. The next three Lemmas establish the competitive and collusive equilibrium in the static game, and show the optimal strategy for a seller that deviates from the collusive agreement. All proofs for this section can be found in Appendix 6.

Lemma 19 (Competitive seller pricing (profit-sharing fees)). *If $\delta < \delta^*$, there is a unique symmetric competitive equilibrium such that in each period t,*

$$p^{com} = c + \tau, \quad \pi^{com}(f) = (1-f)\frac{\tau}{2}, \quad \textit{and} \quad u^{com} = vp^{com} - \frac{\tau}{4}.$$

Lemma 20 (Collusive seller pricing (profit-sharing fees)). *If $\delta > \delta^*$, there is a unique symmetric collusive equilibrium such that in each period t,*

$$p^{com} = v, \quad \pi^{com}(f) = (1-f)\frac{v-c}{2}, \quad \textit{and} \quad u^{com} = -\frac{\tau}{4}.$$

Lemma 21 (Deviating seller pricing (profit-sharing fees)). *If the platform charges a profit-sharing fee and a seller deviates from the collusive agreement in any given period t, the deviating seller charges a price p_t^{dev} and gets a demand d_t^{dev} such that*

$$p_t^{dev} = \frac{p^{col} + p^{com}}{2}, \quad d_t^{dev} = \frac{1}{2} + \frac{p^{col} - p^{com}}{4\tau} \quad \textit{and} \quad \pi_t^{dev}(f) = \frac{1-f}{2\tau}\left(\frac{v-c}{2} + \frac{\tau}{2}\right)^2.$$

In contrast to per-unit fees, seller profits are multiplicative in the profit-sharing fee f. Similar to the additivity of membership fees, the next lemma shows that also the profit-sharing fees' multiplicability prevents the platform from gaining influence on sellers' collusion behavior:

Lemma 22 (Collusion incentives (profit-sharing fees fees)). *The threshold value for sellers' common discount factor $\delta^*(f)$ for future profits to make collusion subgame-perfect is independent of the platform's profit-sharing fee f.*

As the profit-sharing fee is a multiplicable constant in sellers' profits, it cancels out in the incentive compatibility constraint for collusion. Figure 5 depicts the independence of the critical discount factor with the profit-sharing fee.

Since there exists no relationship between the platform's profit-sharing fee f and the critical discount factor δ^*, I conclude that the platform cannot influence seller conduct via its fee. Although the platform may, in principle, set the fee higher than 100 percent, this fee cannot be optimal since it would imply that sellers will charge a price greater than v which eliminates all trades in the marketplace. Hence, a profit-sharing fee of $f > 1$ is always strictly dominated by a fee that is weakly smaller than that (i.e., $f \leq 1$) as long as $f > 0$. Consequently, the platform will operate at a fee $f = 1$ regardless of the sellers' conduct.

A4.3 User entry and platform behavior with profit-sharing fees

When the platform employs profit-sharing fees, the number of sellers is maximized while the number of buyers is minimized. To see this, note that since the platform will charge $f = 1$, sellers will make no profits. Hence, due to the seller entry condition (see Expression (5)), all sellers join, and –by forward induction– they charge a price equal to v. Consequently,

$$N^S(f=1) = 2 \quad \text{and} \quad N^B(p=v) = \frac{1}{1-\delta}\left(a - \frac{\tau}{4}\right). \tag{87}$$

Finally, since $f = 1$ is a dominant choice for the platform, in the first stage it will set its fee to extract all seller profits. Thus, even though the platform cannot influence seller collusion incentives directly with its fee, it can set the fee high enough such that sellers charge the collusive price.

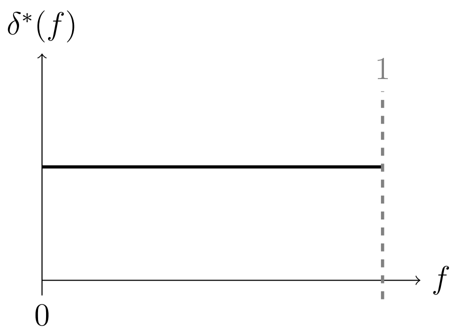

Figure 5: Collusion incentives depending on the fee f with profit-sharing fees.

Appendix 6 Proofs for Appendix 5

Proof of Lemma 19

This proof is analogous to the proof of Lemma 1.

Proof of Lemma 20

This proof is analogous to the proof of Lemma 2.

Proof of Lemma 21

This proof is analogous to the proof of Lemma 3.

Proof of Lemma 22

Proof. After modifying sellers' competitive profits and collusion profits from the previous Lemmas to with the unit costs c, the threshold value for sellers' common discount factor to make collusion subgame-perfect is

$$\delta^*(f) = \frac{\pi^{dev}(f) - \pi^{col}(f)}{\pi^{dev}(f) - \pi^{com}(f)} = \frac{\left(\frac{v-c+\tau}{2}\right)^2 - \tau(v-c)}{\left(\frac{v-c+\tau}{2}\right)^2 - \tau^2}, \tag{88}$$

whose derivative with respect to f is equal to zero and therefore independent of any f. Importantly, it must be that $t \leq v - c$, because else the necessary condition for seller collusion is violated. Therefore, also $\tau^2 \leq \tau(v-c)$, so $\delta^* = 1$ in case of profit-sharing. □

Appendix 7 Revenue-sharing fees

In this section, I extend my analysis to the case where the platform uses royalty fees to employ a pricing scheme based on revenue-sharing. As Table 4 shows, revenue-sharing is a frequently used monetization method for various types of platforms.[26] In particular, I show that my results from Sections 4 and 5 remain qualitatively unchanged when the platform employs revenue-sharing fees instead. I further show that with revenue-sharing, however, the platform's objective function in case of seller competition becomes an irrational function of the fees f such that a subsequent analysis becomes intractable. Nonetheless, given the nature of the fee and how it affects the behavior of both sellers and the platform, I further argue that results on the comparative statics for when the platform is more likely to induce seller collusion should be comparable to the case of per-unit fees.

A6.1 Model with revenue-sharing fees

To accommodate revenue-sharing fees, let's rewrite sellers' unit costs as $c/(1-f) \equiv c'(f)$ instead of $c + f$, as has been done for per-unit fees. Based on this, I next proceed with a description of the sellers' behavior before analyzing users' entry decisions. I conclude this Appendix by showing that in case of revenue-sharing fees, the platform's maximization problem becomes an irrational function whose first-order condition admits a cubic polynomial in case sellers compete. Thus while I can solve for the platform's optimal fee choice given the two distinct types of seller conduct, the analysis for which level of the discount factor δ the platform prefers which type of conduct becomes technically too involved to solve analytically.

A6.2 Seller behavior and collusion incentives with revenue-sharing fees

Following the adjustment, the next lemmas replicate the results of Section 4 for the case of revenue-sharing fees. As for per-unit fees, the platform can encourage seller collusion by charging a larger fee as this makes deviating from the collusive agreement relatively less

Table 4: Examples of platforms using revenue-sharing fees

Business field	Platform and type of revenue-sharing fees
Online marketplaces	Amazon (referral fees), eBay (per order fee)
Short-term rental	Airbnb (royalty fees), Booking.com (booking commission)
Digital content	Patreon (sales fees), Upwork (commission), Fiverr (commission)
Food delivery	Uber Eats (order fee)

[26] The sources of Table 4 can be found at the end of this Appendix.

appealing.

The next three lemmas establish the competitive and collusive equilibrium, respectively, and the deviator's best response to a colluding sellers:

Lemma 23 (Competitive seller pricing (revenue-sharing fees))**.** *If $\delta < \delta^*$ and $f \leq (v - c - \tau)/(v - \tau)$, there is a unique symmetric equilibrium with royalty fees such that*

$$p^{com}(f) = c'(f) + \tau, \quad \pi^{com}(f) = (1-f)\frac{\tau}{2}, \quad \text{and} \quad u^{com}(f) = v - p^{com}(f) - \frac{\tau}{4}$$

in each period t.

Lemma 24 (Collusive seller pricing (revenue-sharing fees))**.** *If $\delta \geq \delta^*$, there is a unique symmetric collusive equilibrium with royalty fees such that*

$$p^{col} = (1-f)\frac{v - c'(f)}{2}, \quad \pi^{col} = (1-f)\frac{v - c'(f)}{2}, \quad \text{and} \quad u^{col} = -\frac{\tau}{4}$$

in each period t.

Lemma 25 (Deviating seller pricing (revenue-sharing fees))**.** *If the platform charges a royalty fee $f \leq (v - c - \tau)/(v - \tau)$ and a seller deviates from the collusive agreement in any give period t, the deviating seller charges a price $p^{dev}(f)$ and gets a demand $d^{dev}(f)$ such that*

$$p^{dev} = \frac{p^{com} + p^{com}(f)}{2}, \quad d^{dev}(f) = \frac{1}{2} + \frac{p^{col} - p^{com}(f)}{4\tau}, \quad \text{and} \quad \pi^{dev} = \frac{1-f}{2\tau}\left(\frac{v - c'(f) + \tau}{2}\right)^2.$$

Based on this, I can rewrite most of the previous results without any major modifications with the new unit cost c'. However, one must exert caution to the result from the last Lemma (on how the threshold value $\delta^*(f)$ reacts with respect to f), as these fees act differently to per-unit fees. The next Lemma as well as Figure 6 illustrate how royalty fees influence sellers' ability to collude:

Lemma 26 (Collusion stability (revenue-sharing fees))**.** *The threshold value for sellers' common discount factor $\delta^*(f)$ for future profits to make collusion subgame-perfect decreases monotonically in the platform's royalty fee f.*

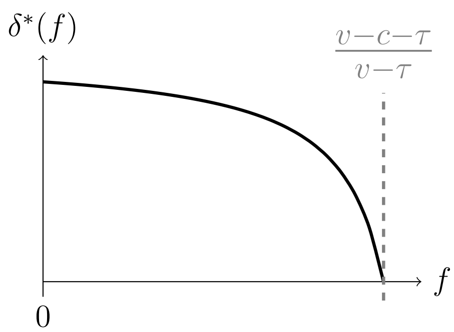

Figure 6: Collusion incentives depending on the revenue-sharing fee f.

A6.3 User entry with revenue-sharing fees

Since the revenue-sharing fee f can be interpreted as a scaling factor for sellers' marginal costs, the next Lemma replicates the findings of Section 5 for revenue-sharing fees.

Lemma 27 (User entry (revenue-sharing fees))**.** *Assume Lemmas 23 and 24 hold. Then, the mass of buyers who enters the platform is given by*

$$N^B(p^{com}(f)) = \frac{1}{1-\delta}\left(v - c'(f) + a - \frac{5\tau}{4}\right) \quad \textit{and} \quad N^B(p^{col}) = \frac{1}{1-\delta}\left(a - \frac{\tau}{4}\right)$$

in case of seller competition and seller collusion, respectively. Moreover, the mass of sellers who enters the platform is $N^S(p^{com}(f)) = N^S(p^{col}) = 2$ *when sellers compete and collude, respectively.*

A6.4 Platform governance with revenue-sharing fees

Building on the results above, I now characterize the platform's optimal fee choices under both seller collusion and seller competition. I then show that deriving the optimal fee as a function of the discount factor δ becomes analytically intractable. However, since revenue-sharing fees influence user behavior in a manner similar to per-unit fees and involve the same fundamental trade-off –maximizing profits through transaction volume or surplus extraction– they are likely to yield comparable results. Thus, at least intuitively, revenue-sharing fees should operate in much the same way as per-unit fees.

Assume first that sellers collude. Then the platform maximizes its discounted profits

$$\begin{aligned} \max_f \Pi^P(p^{col}, f) \; &= \; \frac{1}{1-\delta} * \frac{1}{2} * f * p^{col} * N^B(p^{col}) * N^S(p^{col}) \\ \text{s.t. } \; \delta \; &\geq \; \delta^*(f) \; \text{ and } \; f \leq \frac{v-c-\tau}{v-\tau} \end{aligned} \tag{89}$$

and $p^{col} = v$. Just as for the case of per-unit fees, by Lemma 26 a larger revenue-sharing fee makes collusion for sellers more favorable, so the constraint on δ becomes more relaxed as f increases. Moreover, as the discounted platform profits strictly increase in f when sellers collude, the platform seeks to maximize its fee, setting it equal to the upper bound. I summarize this finding, as well as its comparative statics, in the subsequent two Lemmas:

Lemma 28 (Optimal fee – seller collusion (revenue-sahring fees))**.** *When the platform wants to promote seller collusion, it sets its fee to* $f^{col} = \frac{v-c-\tau}{v-\tau}$ *to extract all surplus.*

Lemma 29 (Comparative statics of Lemma 28)**.** *When sellers collude, the platform's optimal fee* f^{col} *is independent of the stand-alone utility* a*. Moreover,* f^{col} *decreases in the product differentiation parameter* τ*, and increases in the total surplus* $(v-c)$*.*

Conversely, consider now the case in which the platform wants to foster seller competition. Formally, the platform then faces the maximization problem

$$\max_f \Pi^P(p^{com}(f)) = \frac{1}{1-\delta} * \frac{1}{2} * f * p^{com}(f) * N^B(p^{com}(f)) * N^S(p^{com}(f))$$
$$\text{s.t. } \delta \leq \delta^*(f),\ f \leq \frac{v-c-\tau}{v-\tau}, \text{ and } p^{com}(f) = \frac{c}{1-f} + \tau, \tag{90}$$

which admits an irrational objective function of the revenue-sharing fee f. However, since $\Pi^P(p^{com}(f))$ is a continuous and a quasi-concave function of f in the interval $[0, 1]$, a maximum is guaranteed to exist (see Figure 7). Even though the first-order condition becomes a cubic polynomial, the next Lemma illustrates the platform's optimal revenue-sharing fee for very low δ such that the constraint $\delta \leq \delta^*(f)$ does not bind:

Lemma 30 (Optimal fee – seller competition (revenue-sharing fees))**.** *When the platform wants to promote seller competition, there exists a δ small enough such that the optimal per-unit fee is*

$$f^{com} = 1 - \frac{2^{1/3}A}{Q\,\Delta^{1/3}} + \frac{\Delta^{1/3}}{2^{1/3}Q}$$

with $\Delta = B + \sqrt{4A^3 + B^2}$ and

$$A = 8\tau c\left(a^2 + ac + cv + v^2 - 28\tau\,(a+v) + 16av - \frac{45}{2}\tau^2\right),$$
$$B = -48(\tau c)^2\left(72\left(a^2+v^2\right) + 12a\left(12v - 15\tau) + \frac{225}{2}\tau^2\right)\right),$$
$$Q = 3\tau\,(4(a+v) - 5\tau)\,.$$

The result follows the logic of Lemma 7 in the case where δ is sufficiently small. Specifically, when δ is low enough, the constraint on f related to δ does not bind, allowing the platform to set its fee without considering it. As a result, the f^{com} derived in Lemma 30 constitutes an interior solution to the platform's maximization problem under seller competition.

Analyzing the platform's optimal fee under revenue-sharing fees introduces additional complexity. Solving the constrained maximization problem analytically and deriving comparative statics with respect to δ becomes intractable. Nonetheless, since the platform's

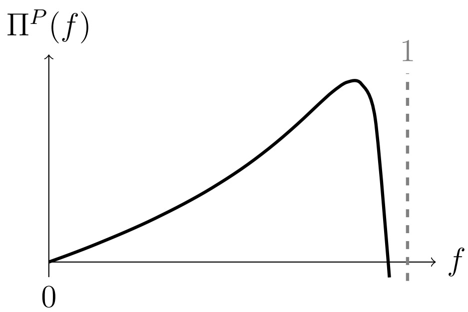

Figure 7: Platform profits under seller competition when unconstrained by $\delta \leq \delta^*(f)$.

revenue channels under revenue-sharing fees mirror those under per-unit fees—either increasing the number of transactions or extracting more surplus per transaction—I conjecture that the comparative statics should be broadly similar.[27] In particular, the platform is more likely to promote seller collusion when the available surplus $(v - c)$ is high and to foster seller competition when network effects are sufficiently strong.

Given this complexity, it is analytically difficult to establish conditions under which the platform's profits under seller competition are greater compared to seller collusion. However, it is easy to show that when the platform is constraint in setting its fee optimally, a larger discount factor δ decreases platform profits:

Lemma 31 (Constraint fee – seller competition (revenue-sharing fees))**.** *When the platform wants to promote seller competition, but is constraint in setting its fee by sellers' collusion incentives, platform profits decrease in δ.*

The reasoning behind the lemma above is very similar to the one for per-unit fees, so I refer to the main text for further explanations. Interestingly, however, this result shows that one can draw platform's profits with respect to the discount factor δ as in Figures 3a and 3b. Therefore, even though I cannot provide a sufficient analysis for comparative statics, the main result of the paper still holds for revenue-sharing fees: if platform profits under seller competition are larger than platform profits under seller collusion, an increased discount factor makes collusion more appealing for the platform. Finally, and based on this, when the platform's unconstrained profits under competition are larger than under seller collusion, one might conjecture that comparative statics in case of revenue-sharing fees might be comparable to those on per-unit fees.

A6.5 Sources for Table 4

Online marketplaces

- **Amazon.** Amazon uses revenue-sharing fees that currently range between 3 percent and 45 percent of sellers' revenues. Source: https://sell.amazon.com/pricing#referral-fees. Visited on January 14, 2025.
- **eBay.** eBay employs per-order fees that act as revenue-sharing fees. Source: https://www.ebay.com/help/selling/fees-credits-invoices/selling-fees?id=4822. Visited on January 14, 2025.

[27] As Lemma 29 indicates, comparative statics may also depend on the product differentiation parameter τ. Given this, I abstract from the effects of τ in the intuition that follows.

- **Etsy**. The online marketplace Etsy charges a revenue-sharing fee (called "transaction fee") of 6.5 percent of all sales. Source: https://www.etsy.com/legal/fees/#fee-types. Visited on January 14, 2025.

Short-term rental

- **Airbnb.** Airbnb currently asks for a commission of about 3 percent per booking. Source: https://www.airbnb.com/resources/hosting-homes/a/how-much-does-airbnb-charge-hosts-288. Visited on January 14, 2025.

- **Booking.com.** Similarly, the website Booking.com also requests a commission that is a percentage share of the booking value. Source: https://partner.booking.com/en-us/help/commission-invoices-tax/commission/understanding-our-commission. Visited on January 14, 2025.

Digital content

- **Patreon.** The website Patreon charges content providers different commissions for each sale, ranging from 5 to 12 percent with three different creator plans. Source: https://support.patreon.com/hc/en-us/articles/11111747095181-Creator-fees-overview. Visited on January 14, 2025.

- **Upwork.** The platform Upwork charges freelancers a commission of about 10 percent for each contract. Source: https://www.upwork.com/tools/fiverr-fee-calculator#:~:text=Fiverr%20charges%20a%2020%25%20service,rate%20you%20charge%20your%20clients. Visited on January 14, 2025.

- **Fiverr.** Similar to Upwork, Fiverr charges a fee of about 20 percent for each transaction. Source: https://www.fiverr.com/start_selling. Visited on January 14, 2025.

Food related

- **Uber Eats.** Uber Eats currently offers three different plans that contain a revenue-sharing fee ranging between 15 and 30 percent. Source: https://merchants.ubereats.com/us/en/pricing/. Visited on January 14, 2025.

Appendix 8 Proofs for Appendix 7

Proof of Lemma 23

Proof. The proof is analogous to the proof of Lemma 1. □

Proof of Lemma 24

Proof. The proof is analogous to the proof of Lemma 2. □

Proof of Lemma 25

Proof. The proof is analogous to the proof of Lemma 3. □

Proof of Lemma 26

Proof. After modifying sellers' competitive profits and collusion profits from the previous Lemmas to with the unit costs of $c' = c/(1-f)$ as a function of f, the threshold value for sellers' common discount factor to make collusion subgame-perfect is

$$\delta^*(f) = \frac{\pi^{dev}(f) - \pi^{col}(f)}{\pi^{dev}(f) - \pi^{com}(f)} = \frac{(v - c'(f) - \tau)^2}{(v - c'(f) + \tau)^2 - 4\tau^2}, \tag{91}$$

whose derivative with respect to f is proportional to $-(v - c' - \tau)$ and therefore negative for any f. □

Proof of Lemma 28

Proof. The proof of Lemma 28 is analogous to the proof of Lemma 5. □

Proof of Lemma 29

Proof. The proof of Lemma 29 is analogous to the proof of Lemma 6. □

Proof of Lemma 30

Proof. Even though the proof of Lemma 30 is mathematically not very evolved, it is quite long and cumbersome. I hence provide only a sketch of it. In fact, assume that δ is sufficiently low such that the constraint $\delta \leq \delta^* f$ does not bind. Then, since $\Pi^P(p^{com}(f))$ is an irrational function, its first order condition with respect to f is a cubic polynomial set equal to zero. Moreover, since $\Pi^P(p^{com}(f))$ is continuous and quasi-concave in f for any $f \in [0, 1]$, a local maximum is guaranteed to exist.

By first depressing the resulting cubic equation from the first-order condition, using Cardano's formula then reveals the roots of the maximization problem. It can be shown that since this cubic equation admits only one root, only one maximum exists, which is the one stated in Lemma 30. This completes the proof. □

Proof of Lemma 31

Proof. Suppose the platform wants to promote seller competition, but it is constraint in setting its fee optimally by sellers' incentives to collude. Then, if the corner solution is $\tilde{f}$,

$$\left.\frac{\partial \Pi^P}{\partial f}\right|_{f=\tilde{f}} > 0, \tag{92}$$

because otherwise, the interior solution would not be optimal anymore, which would be a contradiction. The proof thus consists in showing that

$$\frac{d2(1-\delta)\Pi^P(f^{com})}{d\delta} < 0 \tag{93}$$

for f^{com} being equal to $\tilde{f}$.

To start, note that, as for per-unit fees, it holds by the chain rule that

$$\frac{d2(1-\delta)\Pi^P(f^{com})}{d\delta} = \frac{d2(1-\delta)\Pi^P(f^{com})}{df} * \frac{df}{d\delta} \tag{94}$$

for any f^{com}. As mentioned before, I already established that at $f = \tilde{f}$, $\Pi^P(f^{com})$ is increasing in f^{com}. Hence, I next need to show that $\tilde{f}$ decreases in δ.

To do so, recall that $\tilde{f}$ is such that

$$\delta = \frac{\left(\pi^{col}(f) - \pi^{com}(f)\right)^2}{\left(\pi^{col}(f) + \pi^{com}(f)\right)^2 - 4\left(\pi^{com}(f)\right)^2} = \frac{\left(v - \tau - \frac{c}{1-f}\right)^2}{\left(v + \tau - \frac{c}{1-f}\right)^2 - 4\tau^2}, \tag{95}$$

which can be rearranged such that there are two potential candidates for $\tilde{f}$ to be a constrained optimizer for seller competition:

$$\frac{c}{1-\tilde{f}} = \left\{ v - \tau - 4\tau\frac{\delta}{1-\delta}; v - \tau \right\}. \tag{96}$$

Note that if $c/(1-\tilde{f}) = v - \tau$, then

$$\tilde{f} = \frac{v - c - \tau}{v - \tau} = f^{col}. \tag{97}$$

Hence, if $c/(1-\tilde{f}) = v - \tau$, $\tilde{f}$ induces collusion, contradicting the case that $\tilde{f}$ is a constraint optimizer for seller competition. As a result, it must hold that

$$\frac{c}{1-\tilde{f}} = v - \tau - 4\tau\frac{\delta}{1-\delta}, \tag{98}$$

which yields

$$\tilde{f} = \frac{v - c - \tau - 4\tau\frac{\delta}{1-\delta}}{v - \tau - 4\tau\frac{\delta}{1-\delta}}, \tag{99}$$

which is necessarily between zero and one.

Moreover, taking the derivative of $\tilde{f}$ with respect to δ using the quotient rule reveals that

$$\frac{d\tilde{f}}{d\delta} = -\frac{4\tau c}{\left(v - \tau - 4\tau\frac{\delta}{1-\delta}\right)^2}, \tag{100}$$

which is indeed negative. Therefore, one finally obtains that

$$\frac{d2(1-\delta)\Pi^P(f^{com})}{d\delta} = \underbrace{\left.\frac{d2(1-\delta)\Pi^P(f^{com})}{df}\right|_{f=\tilde{f}}}_{>0} * \underbrace{\frac{df}{d\delta}}_{<0}, \tag{101}$$

so $\Pi^P(f^{com})$ is indeed decreasing in $f^{com} = \tilde{f}$. □

Appendix 9 Other extensions

A8.1 Seller behavior and user entry when the market becomes partially covered in the presence of per-unit fees

I now illustrate how a violation of the "no-local-monopoly" assumption affects seller behavior and users' entry decisions when the platform charges a per-unit fee. In particular, the results in Sections 4 and 5 established sellers' equilibrium behavior and users' entry decisions when the platform fee is low enough, i.e.,

$$f \leq v - c - \tau. \tag{102}$$

In this part, I show that when the market is not fully covered (i.e., Condition (102) fails), sellers act as local monopolists and set prices accordingly. In equilibrium, when the market is fully covered, sellers charge $p^{com} = v$ in each period t. However, when coverage is incomplete, prices drop to $p^{com} = c + f$. The reason is straightforward: for sufficiently high (but not too high) fees (i.e., $v - c - \tau < f \leq v - c$) the standard Hotelling price, $c + f + \tau$, exceeds buyers' willingness to pay, v. Buyers then opt for their outside option rather than purchasing, forcing sellers to cut prices. Since buyers located at the endpoints of the Hotelling line purchase whenever $p \leq v$, it is optimal for sellers to set $p = v$ and capture the entire market. In this case, sellers operate as local monopolists, earning positive profits with a margin of $p - c - f \geq 0$. The platform, in turn, controls these margins directly through f.

When f exceeds $v - c$, however, seller profits turn negative at $p = v$, making it unprofitable to maintain this price. Any price that at least covers costs, $p \geq c + f$, strictly dominates $p = v$ by preventing losses. The result is that sellers set $p = c + f$, earning zero profits. But at this price, buyers stop purchasing altogether, and the platform's market ceases to function.

A8.1.1 Seller behavior when the market is partially covered

I now adjust Lemma 1 to the setting of a partially covered market. To do so, I assume that δ and δ^* are exogenously drawn by nature, and that δ is low enough such that sellers are ensured to compete:

Lemma 32. *When sellers compete and $f > v - c - \tau$, there is a unique symmetric equilibrium such that in each period t,*

$$p^{com} = \begin{cases} v & \text{if } f \leq v - c \\ c + f & \text{if } f > v - c \end{cases}, \quad \pi^{com}(f) = \begin{cases} \frac{v-c-f}{2} & \text{if } f \leq v - c \\ 0 & \text{if } f > v - c \end{cases}, \quad \text{and} \quad u^{com} = -\frac{\tau}{4}$$

As the Lemma above shows, local monopolies exist whenever $f > v - c - \tau$ and $f \leq v - c$. However, when f increases further such that $f > v - c$, the entire marketplace collapses as buyers refrain from purchasing in each period t.

Additionally, note that when sellers collude, they coordinate on playing the same prices as in the competitive equilibrium since any other price decreases profits. As a result, there is no profitable deviation strategy, so the competitive equilibrium and the collusive equilibrium coincide:

Lemma 33. *When sellers collude and $f > v - c - \tau$, there is a unique symmetric equilibrium that is identical to the competitive equilibrium with*

$$p^{col} = p^{com}, \quad \pi^{col} = \pi^{com}, \quad \text{and} \quad u^{col} = u^{com}$$

in each period t. Moreover, there exists no deviation strategy that yields larger per-interaction profits in any period t.

To close this subsection, note that Lemma 33 states that colluding sellers have no incentive to deviate from the collusive agreement as they are indifferent between competing and colluding since $\pi^{col} = \pi^{com}$ in each period t. It therefore follows that the threshold naturally becomes independent of the platform fee f, as $\pi^{dev} = \pi^{col} = \pi^{com}$, so $\delta^* = 0$ for all f. Hence, the platform has no influence on seller behavior with its fee if $f > v - c - \tau$ is imposed.

A8.1.2 User entry decisions when the market is partially covered

Based on these considerations, I next establish that when sellers are local monopolies, the mass of buyers will be minimized, and sellers might be discouraged from joining the platform (as illustrated in Section 5 in the analysis of the collusive equilibrium).

Lemma 34. *Depending on the fee f, the mass of buyers who enters the platform is*

$$N^B(p^{com}) = N^B(p^{col}) = \frac{1}{1-\delta}\left(a - \frac{\tau}{4}\right)$$

in both the competitive and collusive equilibrium. Moreover, the number of sellers who join the platform is

$$N^S(p^{com}(f)) = N^S(p^{col}(f)) = \begin{cases} 2 & \text{if } f \leq v - c \\ 0 & \text{else} \end{cases}.$$

A8.2 Collusion incentives and the Folk Theorem

In this section, I show that the result of Lemma 4 can be extended to any collusive price between the competitive price $p^{com}(f) = c + f + \tau/2$ and the monopoly price $p^{mon} = v$, i.e., to any collusive price $p^{col} \in (p^{com}(f), p^{mon})$ if $f \leq v - c - \tau$.

To see this, note that any collusive price between $p^{com}(f)$ and p^{mon} must be a convex combination between these two, i.e.,

$$\exists\, \alpha \in (0,1) \ \text{ s.t } \ p^{col}(f) = \alpha p^{com}(f) + (1-\alpha)p^{mon}. \tag{103}$$

Then, by Lemma 3, the best response of seller i to a seller $-i$ who charges p^{col} becomes

$$p_t^{dev}(f) = \frac{p^{col}}{2} + \frac{p^{com}(f)}{2} = \frac{p^{mon} + p^{com}(f)}{2} - \alpha\frac{p^{mon} - p^{com}(f)}{2}$$
$$= \frac{v}{2} + \frac{c+f+\tau}{2} - \alpha\frac{v-c-f-\tau}{2}, \quad (104)$$

which is less than the deviation price in Lemma 3. The next Lemma adjusts Lemma 3 to the new collusive price:

Lemma 35 (Deviating seller pricing – Folk Theorem)**.** *If $f \leq v - c - \tau$ and colluding sellers agree to fix their price to Expression (103), then a seller who deviates from the collusive agreement in any given period t sets a price p_t^{dev} and gets a profit π_t^{dev} such that*

$$p_t^{dev}(f) = \frac{p^{mon} + p^{com}(f)}{2} - \alpha\frac{p^{mon} - p^{com}(f)}{2}$$
$$\text{and} \quad \pi_t^{dev}(f) = \frac{1}{4\pi^{com}}\left(\pi^{com} + \alpha\pi^{com} + (1-\alpha)\pi^{mon}(f)\right)^2.$$

Therefore, if sellers collude on a price below the monopoly price v, a deviating seller chooses a price that is even lower than the collusive price and thus gets a greater profit. Note, however, that the deviation profit in Lemma 35 is lower than the deviation profit in Lemma 3 since also the deviation price is lower.

Next, I show that even with this deviation price, the threshold discount factor $\delta^*(f)$ decreases in f:

Lemma 36 (Collusion incentives – Folk Theorem)**.** *When $p^{col(f)}$ as in Expression (103), the threshold value for sellers' common discount factor $\delta^*(f)$ for future profits to make collusion subgame-perfect decreases monotonically in the platform's per-unit fee f.*

To understand the result above, recall that Lemma 35 shows that –similar to Lemma 3– a deviating seller undercuts its colluding rivals and thus gets a greater market share which in turn leads to greater profits. However, since the collusive price is a convex combination between the competitive price $p^{com(f)}$ and the monopoly price v, also deviating price price has to be a convex combination that is lower than the collusive price. As a result, and compared to Lemma 3, deviating from the collusive agreement becomes even less attractive. Thus, by raising the per-unit fee f, the platform can render deviations even less attractive. Therefore, even for a collusive price below the monopoly price, Lemma 36 shows that collusion incentives increase when the platform increases its fee.

A8.3 Collusion incentives and scale-invariant demand

To demonstrate that the result of Lemma 4 (for per-unit or revenue-sharing) is not confined to the Hotelling model, I develop a stylized, yet more general model of horizontal differentiation as in the spirit of Singh and Vives (1984) by keeping the timeline unchanged to the baseline model from Section 3. Importantly, I use a stylized Singh and Vives (1984) model to show that larger platform fees increase collusion incentives as long as demand features a property called scale invariance. I moreover advocate that this property is met in all models that use market shares instead of absolute demand.

To start, let the demand of a seller i in any period t be defined as

$$q_t^i(pi_t, p_t^{-i}) = v - p_t^i + \beta p_t^{-i}, \tag{105}$$

where $v > 0$ is the choke price, p_t^i and p_t^{-i} are the prices of seller i and $-i$ in period t, respectively, and $0 \leq \beta < 1$ measures the degree of substitutability between product i and $-i$. Note that a seller's market share in any given period is defined as

$$(d_t^i)^S = \frac{q_t^i}{q_t^i + q_t^{-i}}, \tag{106}$$

and that maximizing one-period profits based on the market share is therefore equivalent to total profit maximization based on the total demand.[28] Consequently, in any period of the third stage, seller i's maximization problem reads

$$\max_{p_t^i} \pi_t^i = (p_t^i - c - f) * (d_t^i)^S. \tag{107}$$

Finally, note that as $d_t^i \in [0, 1]$, one can rewrite $(d_t^i)^S$ in the form of the standard Hotelling demand:

$$(d_t^i)^S = \frac{1}{2}(1 + x) \quad \text{where} \quad x = \frac{q_t^i - q_t^{-i}}{2(q_t^i + q_t^{-i})}.$$

Therefore, i's market share $(d_t^i)^S$ is a positive affine transformation of the Hotelling demand from the baseline model in Section 3.

Based on this observation, the next Lemma establishes first that both the Hotelling demand and a "market share"-model exhibit an important property, namely scale invariance (or homogeneity of degree zero), that implies the threshold discount factor to sustain collusion δ^* to be monotonically decreasing in the platform's fee f:

Lemma 37 (Scale invariance and implications for the threshold discount factor)**.** *Define the Hotelling demand as*

$$(d_t^i)^H(p_t^i, p_t^{-i}, \tau) = \frac{1}{2} + \frac{p_t^{-i} - p_t^i}{2\tau},$$

[28] Indeed, a stronger focus on market shares instead of absolute demand can be particularly advantageous for short-term profit maximization: By capturing a larger share of the market, a seller can achieve economies of scale and enhance its bargaining power with suppliers. Additionally, a higher market share can deter potential entrants, thereby reducing competitive pressures and allowing sellers to maintain higher prices and profits in the short run. As such, in most strategic settings a focus on market shares might be more realistic than absolute demand.

where p_t^i and p_t^{-i} are the prices set by firms i and $-i$ and $\tau > 0$ is a product differentiation parameter. Define further the Singh and Vives (1984) market share as

$$(d_t^i)^S(v, p_t^i, p_t^{-i}) = \frac{q_t^i}{q_t^i + q_t^{-i}}$$

with v is a reservation value and $0 \leq \beta < 1$. Then

1. *Both $(d_t^i)^H(p_t^i, p_t^{-i}, \tau)$ and $(d_t^i)^S(v, p_t^i, p_t^{-i})$ are scale-invariant: for any $\alpha > 0$,*

$$(d_t^i)^H(\alpha\, p_t^i, \alpha\, p_t^i, \alpha\, \tau) = (d_t^i)^H(p_t^i, p_t^i, \tau) \quad \text{and} \quad (d_t^i)^S(\alpha\, v, \alpha\, p_t^i, \alpha\, p_t^i) = (d_t^i)^S(v, p_t^i, p_t^i).$$

2. *In a symmetric equilibrium (i.e., when $p_t^i = p_t^{-i}$) both functions yield a market share of $1/2$. Moreover, as only relative prices matter (due to scale invariance), the effects of a change in the platform fee f on the collusive, competitive, and deviation profits enter in an identical manner in both the Hotelling and Singh and Vives (1984) specifications. Consequently, the threshold discount factor δ^* monotonically decreases in f in both models.*

The scale invariance property implies that for both types of market shares only relative prices matter. Thus, a proportional scaling of the overall market size leaves the structure of demand –and hence the equilibrium outcomes such as prices and quantities– unchanged. Consequently, an increase in the platform's fee f will be identical in both models: an increase in f lowers π^{col} more than π^{com}. Importantly, as increases the difference between π^{dev} and π^{col}, it implies that the threshold discount factor δ^* monotonically decreases in f in as long as the demand satisfies scale invariance. Ultimately, this implies that all models that use market shares instead of "absolute-quantities" demand exhibit scale invariance. Hence, this mechanism remains robust for a broad class of economic models.

Appendix 10 Proofs for Appendix 9

Proof of Lemma 32

Proof. Since $\delta < \delta^*$, sellers compete. As buyers' outside option to buy from a seller via the platform is located at the sellers' locations, note that buyers purchase if and only if

$$v - p^{com} \geq 0. \tag{108}$$

Therefore, in principle, sellers would set a price equal to

$$p^{com} = c + f + \tau \tag{109}$$

each period t in equilibrum as long as $v \geq c + f + \tau$ to sell their products.

Consider first the case where f is such that $v - c - \tau < f \leq v - c$. Then, since $f > v - c - \tau$, it follows that $p^{com} = c + f + \tau > v$, so this price cannot be optimal. Consequently, it is optimal for the sellers to charge a price $p^{com} = v$, making buyers indifferent between purchasing and realizing the outside option. sellers then make a per-interaction profit of $\pi^{com} = (v - c - f)/2$. Moreover, as buyers are indifferent between buying and not buying, their per-interaction benefit of $u^{com} = -\tau/4$.

Suppose now that $f > v - c$. Then, if sellers offer their products at a price equal to v, some consumers may buy, so they would generate a loss. Thus, a price equal to v cannot be optimal if $f > v - c$, so sellers must charge a greater per-period price in equilibrium. Moreover, if $p^{com} = c + f$ sellers break even, but cannot generate any positive per-interaction profits in any period t, so $\pi^{com} = 0$ if $f \geq v - c$. Finally, since buyers do not make purchases, their per-period per-interaction benefit is equal to $-\tau/4$. □

Proof of Lemma 33

Proof. The first part of the proof is analogous to the proof of Lemma 32. Hence, also the per-period prices and per-interaction profits and benefits are identical for each period t.

To see that there exists no profitable deviation strategy, recall that any price below p^{col} yields negative profits. Finally, any price greater than p^{col} weakly decreases demand, so any other price is weakly dominated by that. □

Proof of Lemma 34

Proof. Concerning the number of buyers who join the platform in equilibrium, note that in both situations (seller competition and collusion), sellers charge a price equal to v or greater. Since buyers' net utility from purchasing is in both cases zero (since they buy if and only if

$v \geq p$ with $p \in \{p^{com}, p^{com}\}$ or realize the outside option when $v < p$ with $p \in \{p^{com}, p^{com}\}$ which yields zero utility), the number of buyers becomes

$$N^B(p^{com}) = N^B(p^{col}) = \frac{1}{1-\delta}\left(a - \frac{\tau}{4}\right). \tag{110}$$

Regarding the number of sellers, recall that from Corollary 1, that $N^S(p = v, f) = 1$ if $f \leq v - c$ and $N^S(p = v, f) = 0$ else for any $p \in \{p^{com}, p^{col}\}$. Since this remains valid for the case of $f > v - c - \tau$, it concludes the proof. □

Proof of Lemma 35

Proof. This proof is analogous to the proof of Lemma 3. □

Proof of Lemma 36

Proof. Note first that

$$\delta^*(f) = \frac{\pi^{dev}(f) - \pi^{mon}(f)}{\pi^{dev}(f) - \pi^{com}}, \tag{111}$$

so the derivate of $\delta^*(f)$ with respect to f is proportional to

$$\frac{d\left[\pi^{dev}(f) - \pi^{mon}(f)\right]}{df} * \left(\pi^{dev}(f) - \pi^{com}\right) - \frac{d\left[\pi^{dev}(f) - \pi^{com}\right]}{df} * \left(\pi^{dev}(f) - \pi^{mon}(f)\right). \tag{112}$$

Since $\pi^{mon}(f) = (v - c - f)/2$, its derivative with respect to f is $-1/2$. Also, note that $\pi^{dev}(f)$ decreases in f. Denote ther derivative of $\pi^{dev}(f)$ with respect to f by $-A$ with $A > 0$. Then,

$$\frac{d\delta^*}{df} \propto -A * \pi^{dev}(f) - \frac{1}{2}\pi^{dev}(f) + A\pi^{com} + \frac{1}{2}\pi^{com} + A * \pi^{dev}(f) - A * \pi^{mon}(f), \tag{113}$$

which is equal to

$$-\frac{1}{2}\left(\pi^{dev}(f) - \pi^{com}\right) - A\left(\pi^{mon}(f) - \pi^{com}\right). \tag{114}$$

Note that since $f \leq v - c - \tau$, $\pi^{mon}(f) > \pi^{com}$, so also $\pi^{dev}(f) > \pi^{com}$. As a result, Expression (114) must be negative, which implies that $\delta^*(f)$ decreases in f. This completes the proof. □

Proof of Lemma 37

Proof. I first show that both the Hotelling demand and the Singh and Vives (1984) market share are scale-invariant. Based on this, I then proceed to demonstrate that scale invariance induces the threshold discount factor δ^* to sustain collusion to decrease in the platform fee f.

1. *Scale invariance the market shares.*

   Let me first prove that the Hotelling demand is scale-invariant. I then repeat this proof for the Singh and Vives (1984) market share.

(i) Hotelling Demand: For any $\alpha > 0$,

$$(d_t^i)^H(\alpha\, p_t^i, \alpha\, p_t^{-i}, \alpha\, \tau) = \frac{1}{2} + \frac{\alpha\, p_t^{-i} - \alpha\, p_t^i}{2\,(\alpha\, \tau)} = \frac{1}{2} + \frac{p_t^{-i} - p_t^i}{2\tau} = (d_t^i)^H(p_t^i, p_t^{-i}, \tau). \quad (115)$$

(ii) Singh and Vives (1984) market share: Notice that

$$q_t^i(\alpha\, v, \alpha\, p_t^i, \alpha\, p_t^{-i}) = \alpha\, v - \alpha\, p_t^i + \beta\, \alpha\, p_t^{-i} = \alpha\,(v - p_t^i + \beta\, p_t^{-i}) = \alpha q_t^i(v, p_t^i, p_t^{-i}), \quad (116)$$

so by symmetry, also $q_t^{-i}(\alpha v, \alpha p_t^i, \alpha p_t^{-i}) = \alpha q_t^{-i}(v, p_t^i, p_t^{-i})$. This implies that

$$\begin{aligned}(d_t^i)^S(\alpha\, v, \alpha\, p_t^i, \alpha\, p_t^{-i}) &= \frac{\alpha\,(v - p_t^i + \beta\, p_t^{-i})}{\alpha\,(v - p_t^i + \beta\, p_t^{-i}) + \alpha\,(v - p_t^{-i} + \beta\, p_t^i)} \\ &= \frac{v - p_t^i + \beta\, p_t^{-i}}{(v - p_t^i + \beta\, p_t^{-i}) + (v - p_t^{-i} + \beta\, p_t^i)} = (d_t^i)^S(v, p_t^i, p_t^{-i}). \quad (117)\end{aligned}$$

2. *The threshold discount factor to sustain collusion monotonically decreases in f.*

   In a symmetric equilibrium, where $p_t^i = p_t^{-i}$, both $(d_t^i)^H$ and $(d_t^i)^S$ yield a market share of $1/2$. Since both demand functions are scale-invariant and thus homogeneous of degree zero, they depend only on the relative differences in prices. This means that in the evaluation of per-period profits –whether under competition, collusion, or a one-shot deviation– the effect of an increase in f will be identical in both models. In particular, when computing the deviation profit π^{dev}, the collusive profit π^{col}, and the competitive profit π^{com}, the changes in f affect all three in a manner that preserves the relationship

$$\delta^* = \frac{\pi^{dev} - \pi^{col}}{\pi^{dev} - \pi^{com}}. \quad (118)$$

   Since an increase in f increases the temptation to deviate (by lowering π^{col} more than π^{com} and increasing the gap $(\pi^{dev} - \pi^{col})$, it follows that

$$\frac{\partial \delta^*}{\partial f} < 0. \quad (119)$$

   Thus, the comparative statics regarding f are identical in both the Hotelling and the Singh and Vives (1984) model.

□

Appendix 11 Sources for Table 1

I here present the sources of Table 1. In particular, I list for each element of Table 1 its sources (websites) and when I accessed them.

Online marketplaces

- **Amazon: Individual selling plan.** Amazon currently offers an individual selling plan that contains a per-unit fee. Source: https://sell.amazon.com.be/en/tarifs. Visited on January 14, 2025.
- **eBay: Final value fee.** The online marketplace eBay currently charges final value fees that contain a per-unit fee as a per-order fee. Source: https://www.ebay.com/help/selling/fees-credits-invoices/selling-fees?id=4822. Visited on January 14, 2025.

Food delivery

- **Uber Eats.** In some countries, Uber Eats charges merchants a processing fee for each order made via the platform. Source: https://help.uber.com/merchants-and-restaurants/article/pricing?nodeId=bfa4697f-b39f-48f0-985e-757a6c5a8281. Visited on January 14, 2025.
- **DoorDash.** DoorDash is the largest food delivery platform in the US. Together with a revenue-sharing fee, it levies a per-unit fee for every order. Source: https://merchants.doordash.com/en-ca/learning-center/delivery-commission. Visited on January 14, 2025.

Event sales

- **Universe.** The online ticket platform Universe charges an additional per-unit fee for each ticket sold, while the amount of the fee varies with different selling plans. Source: https://www.universe.com/pricing. Visited on January 14, 2025.
- **Brown Paper Tickets.** Currently, Brown Paper Tickets charges a per-unit fee in addition to a revenue-sharing fee for each ticket sold. Source: https://help.brownpapertickets.com/hc/en-us/articles/360023967871-What-is-the-Service-Fee. Visited on January 14, 2025.

Print-on-demand

- **Printful.** Printufl provides certain branding options that enter the final product's price as a per-unit fee. Source: https://www.printful.com/pricing. Visited on January 14, 2025.
- **Printify.** As of now, Printify charges fees for each item sold. Source: https://help.printify.com/hc/en-us/articles/24637007039121-What-is-the-Profit-Calculator. Visited on January 14, 2025.